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Introduction

MBA 840 Project Evaluation is a semester course work of two credit units. It will be available to all students taking the MBA programme in the School of Business and Human Resources Management.

The course consists of 3 modules involving the subject area of Project Evaluation.

The course guide tells you what the course is all about and the relevant materials that you require to make your study very successful. Other vital information contained in this course guide deals with Assessment which consists of the Tutor- Market Assignments, and written examination.

The Course Contents

The course contents consist of project evaluation from the project cycle to engineering evaluation, demand/market evaluation. It also includes financial and economic analyses of projects.

Course Aims

The aims of this course are to expose you to the knowledge of how to evaluate projects either as a project initiator or an evaluator. It aims to sharpen your skills in the evaluation of either new projects or existing projects with a view to deciding whether they meet certain predetermined investment criteria.

Course Objectives

At the end of this course you should be able to:

- discuss the totality of project evaluation
- prepare simple project plans/business plans
- evaluate projects submitted to you for consideration

The Course Materials

The main components of the course are:

- 1. The Course Guide
- 2. Study Units
- 3. References/Further Readings
- 4. Assignments

Study Units

There are 15 units in this course and they should be studied carefully

Module 1

Unit 1	The Project Cycle
Unit 2	Project Evaluations – An Introductory Format
Unit 3	Factors Affecting Location of Projects
Unit 4	Capacity and Production Planning
Unit 5	The Concept Of Engineering Evaluation

Module 2

Unit 1	Manpower Planning and Evaluation
Unit 2	Demand Analysis
Unit 3	Supply Analysis
Unit 4	Competition and Marketing Plans
Unit 5	Project Cost Analysis

Module 3

Unit 1	Projected Income Statement
Unit 2	Projected Cash Flow Statements
Unit 3	Projected Balance Sheets
Unit 4	Project Evaluation Criteria
Unit 5	Introduction to Economic Analyses

Units 1 to 3 provide you with the necessary background knowledge you require for your study. The remaining units 4 to 15 focus attention on the subject matter of project evaluation.

Each study unit will take at least two hours and it includes the introduction, objectives, main content, exercise, conclusion, summary, references and the Tutor-Marked Assignments (TMAs).

You are required to study the materials, reflect on them and do the exercises. Some of the exercises require that you visit some organisations and find out how they carry our project evaluation in practice. You should also read the textbooks and other recommended materials.

Assignments

In each unit, you will find exercises which you are required to do. The exercises will enable you to have a better understanding of what you have studied.

Tutor-Marked Assignment

In doing the Tutor-Marked Assignments, you are expected to apply what you have learnt in the contents of the study unit. The assignments are expected to be turned in to your tutor for grading. They constitute 30% of the total score.

Final Examination and Grading

At the end of the course, you will write the final examination. It will attract the remaining 70%. This makes the final score to be 100%.

Summary

The course MBF 739 – Project Evaluation will expose you to the knowledge and understanding of how to evaluate projects. When you complete the course, you would have been armed with the necessary knowledge required to evaluate projects.

COURSE GUIDE

MBA 816 BASIC MATHEMATICS AND STATISTICS FOR MANAGERS

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INTRODUCTION

The Course, Introduction to Mathematics and Statistics (MBA 816) is a core course which carries two credit units. It is prepared and made available to all the students who are taking the Postgraduate Diploma Programme; a programme tenable in the School of Business and Human Resources Management. The Course is a useful material to you in your academic pursuit as well as in your workplace as managers and administrators.

WHAT YOU WILL LEARN IN THIS COURSE

The Course is made up of fifteen units, covering areas such as the introduction to basic mathematical tools of number system. This introduces you to the basic use of numbers and their applications in mathematical analysis, simple fractions, exponents and roots, analysis of ratios, analysis of variation, elementary treatment of simultaneous equation, quadratic equation, as well as arithmetic and geometric progressions. The remaining part of the course is devoted to statistics, which examines statistical investigation and data collection, data presentation, measures of central tendency, measures of dispersion, analysis of correlation and regression.

This course guide is meant to provide you with the necessary information about the course, the nature of the materials you will be using and how to make the best use of the materials towards ensuring adequate success in your programme as well as the practice of mathematics and statistics. Also included in this course guide are information on how to make use of your time and information on how to tackle the tutor-marked assignments (TMAs). There will be tutorial sessions during which your instructional facilitator will take you through difficult areas and at the same time you will have meaningful interactions with your fellow learners.

The course consists of the basics of mathematics and statistics. The mathematics segment include number system, this introduces you to the basic use of numbers and their applications in mathematical analysis, simple fractions, exponents and roots, analysis of ratios, analysis of variation, elementary treatment of simultaneous equation, quadratic equation, as well as arithmetic and geometric progressions. The remaining part of the course is devoted to statistics, which examines statistical investigation and data collection, data presentation, measures of central tendency, measures of dispersion, analysis of correlation and regression.

COURSE AIMS

The main aim of this course is to expose you to the nature of mathematics and statistics, the mechanisms necessary for using mathematics and statistics in matters within the organisation and the role of mathematics in solving complex problems in daily life. The course also aims at making you have greater appreciation of the roles of mathematics and statistics in resolving many issues in life, business and organisation.

The aims of the course will be achieved in the following ways:

- Explaining the nature of mathematics and statistics;
- Describing the necessary mechanisms and framework for managing mathematical variables, numbers, fractions, exponents;
- Explaining the methods and styles of using simple and simultaneous equations and graphs;
- Describing the necessary strategies for using ratios in mathematical analysis;
- Discussing the nature of variation and its application in business;
- Explaining the nature of progressions and its application;
- Explaining the methods of data collection;
- Identifying and explaining the steps for managing the data collected for statistical analysis; and
- Discussing the peculiar role of correlation and regression as a means of making comparison and forecasting.

COURSE OBJECTIVES

At the end of this course, you should be able to:

- discuss the nature of mathematics and statistics:
- identify the necessary mechanisms for managing mathematical variables like numbers, fractions, exponents, ratios etc;
- explain the mechanisms for solving linear and simultaneous equations in mathematics;
- explain the nature and method of solving problems of arithmetic and geometric progression.
- analyse the various forms of data collection and data analysis in statistics;
- identify the use of measures of central tendency and measures of dispersion;
- describe the strategic role of sampling in statistical investigation;
- discuss the nature of pie charts and bar charts in data presentation and
- explain the use of correlation and regression.

COURSE MATERIALS

Major components of the course are as follows.

- 1. Course Guide
- 2. Study Units
- 3. Textbooks
- 4. Assignment Guide

STUDY UNITS

There are four modules of fifteen units in this course. They should be studied carefully. They are as follows.

Module 1

Unit 1	Number System
Unit 2	Simple Fractions
Unit 3	Exponents and Roots
Unit 4	Ratios
Unit 5	Analysis of Variation

Module 2

Unit 1	Linear Equation
Unit 2 Unit 3	Simultaneous Linear Equation Quadratic Equation
Unit 4	Analysis of Progressions

Module 3

Unit 1	Statistical Investigation and Data Collection
Unit 2	Data Presentation in Statistics
Unit 3	Measures of Central Tendency
Unit 4	Measures of Dispersion

Module 4

Unit 1	Analysis of Correlation
Unit 2	Analysis of Regression

Each study unit will take at least two hours. Each unit includes the introduction, objectives, main content, self-assessment exercises, conclusion, summary and tutor-marked assignment and references/further reading. Some of the self-assessment exercises will necessitate discussion with some of your colleagues. You are advised to

do so in order to practise and become self sufficient in mathematical and statistical issues.

There are also textbooks under the references/further reading section. They are meant to give you additional information if only you can lay your hands on them. You are advised to practise the self-assessment exercises and tutor-marked assignments for greater understanding of the course. By so doing, the stated learning objectives of the course will be achieved.

ASSIGNMENT FILE

There are two kinds of assignments in this course and you are expected to do them by following the schedule prescribed for them in terms of when to attempt them and submit same for grading by your tutor.

TUTOR-MARKED ASSIGNMENT

In doing the tutor-marked assignment, you are to apply the knowledge you have learnt in the contents of the study units. These assignments, which are many in number are expected to be turned in to your tutor for grading. They constitute 30% of the total score for the course

FINAL WRITTEN EXAMINATION

At the end of the course, you will write the final examination. It will attract the remaining 70%. This makes the total final score to be 100%.

CONCLUSION

The course, Basic Mathematics and Statistics for Managers (BHM 811), exposes you to the issues involved in mathematics and statistical methods, and how to practise them. On the successful completion of the course, you would have been armed with the materials necessary for efficient and effective use in mathematics and statistical analysis.

MAIN COURSE

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MODULE 1 INTRODUCTION TO NUMBERS AND THEIR APPLICATIONS

Unit 1	Number System
Unit 2	Simple Fractions
Unit 3	Exponents and Roots
Unit 4	Ratios
Unit 5	Analysis of Variation

UNIT 1 NUMBER SYSTEM

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Natural Numbers
 - 3.2 Whole Numbers
 - 3.3 Integers
 - 3.4 Rational Numbers
 - 3.5 Irrational Numbers
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

Number is one of the foundation concepts in mathematics and it is quite different in concept with numerals. Numerals are signs that serve as the means of representing numbers.

Number system generally is a technique of representing numbers by means of symbols. Modern number systems are value systems by which an individual number value is determined in daily activities of life.

The history of number and numeration is as old as human history and civilization. At early civilisation, people used strokes, pebbles, or notches as the means of measuring the number of goods.

This is done by making strokes on walls, trees, stones or the notches made on a piece of wood to show the number. The process in which objects in one group are represented and compared with those of another group is called matching. The process of matching is also known as tallying. This tallying system of counting is still in use today. There are

different ways that people in different communities use in counting, but what is common to every community is number. Number measures quantity and value and these remain the same all over the world.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the term number
- discuss natural numbers.
- convert numbers to other bases
- explain whole numbers
- explain integers
- discuss rational numbers
- explain irrational numbers.

3.0 MAIN CONTENT

3.1 Natural Numbers

Natural numbers are counting numbers that have definite beginning but no ending. The nature of natural numbers is said to be discrete. They are usually referred to as ordinal numbers. When they denote order, the order should be in magnitude, showing a unique pattern of increase or decrease in arrangement at any given time. Anytime the natural numbers are used to show quantities such as 5 students, 4 cows, 17 cups, they are known as cardinal numbers. Natural numbers have some properties that make it unique. Some of the properties are as listed below.

Counting numbers

They are used for counting in any community or language. The process of counting is often done in various groups, for example, in groups of 2_s, 5_s, 10_s 12_s or 20_s. These number groups form the number bases used for calculations. Some of the groups include *even*, *odd*, *prime*, *square and cubic numbers*. Even numbers are natural numbers divisible by 2, they are numbers like 2,4,6,8,10,12 while *odd numbers* are natural numbers that are not divisible by two. Examples are 1,3,5,7,9,11,13. *Prime Numbers* are natural numbers with no factor other than one or itself, these include numbers like 3,5,7,11,13,15. *Square numbers* are squares of natural numbers raised to the second power and cubic numbers are numbers that are third power of natural numbers. Examples are 4,9,16,25 and 8,27,64 respectively.

Conversion of numbers to other bases

Traditionally, numbers can be converted from one base to another using different methods and techniques. The most common conversion is usually from base 10 to other bases through continuous division with the base in question and expressing the remainder as the digits of the required base in some definite order.

Example, change 86 to base 2

- 2 86
- 2 43 R 0
- 2 21 R 1
- 2 10 R 1
- 2 5 R O
- 2 2 R 1
- 2 1 R 0
- $86_{10} = 10110_2$

The rule is to start expressing the digits from the remainders, beginning from the bottom to the first one i.e. 10110_2 .

3.2 Whole Numbers

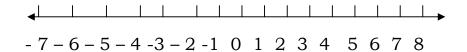
Given that natural numbers are counting numbers beginning from one and the numbers continue without any limit, there were problems that became unresolved such as 2 minus 2, 3-3, and 9-9. The discovery of zero in 600 AD helped to resolve the problems in numbers. This expanded the operation of number system. When zero is added to a set of natural numbers we have what is called whole numbers.

SELF-ASSESSMENT EXERCISE

Explain the term whole number.

3.3 Integers

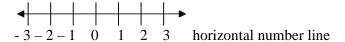
Integers are whole numbers that do not have any form of fraction associated with them. An integer is a combination of positive and negative numbers together with zero. The positive numbers are usually called positive integers; the negative numbers are called negative integers, while the positive and negative numbers are called direct numbers. In mathematical analysis, direct numbers can be represented on a number line.



Conventionally positive integers are a set of natural numbers that are They are written without attaching the positive sign before any of the numbers. However, negative integers are written with the negative sign attached before them or on top of each number distinguishing the negative integers from the positive integers. The only integer that is neither positive nor negative is zero.

3.3.1 Number Line

A number line is a straight line that shows the ordering property and position of integers. The line is made up of arrows ending at one or both sides of the line indicating continuity in the numbers or integers. The number line is divided into equal parts to indicate the position of the integers. Usually, only a small section of the integers can be represented on the line at a time. There are two strategies of drawing the number line. It can be drawn horizontally or vertically. The ordering property of integers is that numbers to the right of the line are always greater than those on the left. Equally those to the left of zero are always less than those to the right of zero. The numbers are usually written in ascending or descending order. When they are in ascending order, they increase from left to right. The signs used to show "greater than or less than" are ">" (greater than) "<" (less than). 5>4 means 5 is greater than 4 and 6 < 7 means 6 is less than 7.



3.3.2 Addition, Subtraction and Multiplication of Integers

In the addition of integers, we count the positive numbers by moving to the right hand side or upwards, while the counting of negative numbers is by moving to the left of zero or downwards.

When subtracting numbers, the following points should be noted.

a) If a positive integer is subtracted or taken away from a smaller positive integer, the answer is always a negative number.

Example 1

$$4 - 6 = -2$$
, $13 - 17 = -4$ etc.

b) In order to subtract a negative integer from another negative integer, we add the absolute value of the negative integers.

Example 2,

$$10 - (-6) = 16 \text{ or } -10 - -15 = 5.$$

Example 3, evaluate the following

$$(a) - 20 - (-24)$$

$$c) - 13 - 16$$

Solution

a)
$$-20 - (-24) = -(20) + 24 = 4$$

b)
$$12 - (-12) = 12 + 12 = 24$$

c)
$$-13 - 16 = -29$$

In the division of integers, when integers are divided together two like signs give a positive result, while two unlike signs give a negative result.

Example 4

$$(+15) \div (+3) = +5$$

$$(-15) \div (-3) = +5$$

$$(-15) \div (+3) = -5$$

$$(+15) \div (-3) = -3$$

It should be noted that any number that is multiplied by zero equals zero, similarly a zero multiplied by any integer equals zero.

SELF-ASSESSMENT EXERCISE

- i. Evaluate the followings.
 - a) -9 (-12)
 - b) 28 (-28)
 - c) $+14 \div (-2)$
 - d) $-8 \div (-4)$.

3.4 Rational Numbers

A rational number is an expression of a ratio of two whole numbers. It can take the form of $^{V}/_{Z}$ or $V \div Z$ where V and Z are integers and Z is not equal to zero at any time. A set of rational numbers X include the set of integers as well as positive and negative fractions. Therefore, the set of integers is a proper subset of the rational numbers. Examples are; 2/4, 1/5, 12/3, 7, 81/8, - 1/3 etc.

The scope of rational numbers has no end in both positive and negative numbers and also within each numbers gap. Example between 0 and 1, 1 and 2, 2 and 3, 0 and -1, -3 and -4

SELF-ASSESSMENT EXERCISE

- i. What are rational numbers b) illustrate examples of rational numbers.
- ii. express the following rational numbers in
 - a) ascending order $-\frac{1}{2}$, -3, 4, 2, $-\frac{3}{4}$
 - b) descending order -11, 9, -4 17, 12, 3

3.5 Irrational Numbers

They are numbers that cannot be written as exact fractions nor expressed as terminating decimals. Irrational numbers usually do not have exact values. Usually irrational numbers which are expressed in the form of roots are known as surds.

Example 5
$$\sqrt{2}$$
, $\sqrt{3}$, $\sqrt{7}$

It should be noted that some numbers which are expressed in form of roots and have exact terminating decimals are rational numbers and do not fall in the category of irrational numbers, example $\sqrt{4}$ $\sqrt{9}$ etc.

When two or more surds are to be multiplied together they should first be simplified. Whole numbers should be taken with whole number and surds with surds.

Example 6 Simplify
$$\sqrt{27} \times \sqrt{50}$$

= $\sqrt{(9 \times 3)} \times \sqrt{(25 \times 2)}$
= $3 \sqrt{3} \times 5 \sqrt{2}$
= $15\sqrt{6}$

Example 7 Multiply the irrational numbers (surd)

$$\sqrt{12} \quad x \quad 3\sqrt{60} \quad x \quad \sqrt{45} \\
= \quad \sqrt{(4 \times 3)} \quad x \quad 3\sqrt{(4 \times 15)} \quad x\sqrt{(9 \times 5)} \\
= \quad 2\sqrt{(3)} \quad x \quad 3 \quad x \quad 2\sqrt{15} \quad x \quad 3\sqrt{5} \\
= \quad 36 \quad \sqrt{(3 \times 15 \times 5)} \\
= \quad 36 \quad x \quad 15 \\
= \quad 540.$$

Example 8. Simplify
$$3\sqrt{50} - 5\sqrt{32} + 4\sqrt{8}$$

= $3\sqrt{(25 \times 2)} - 5\sqrt{(16 \times 2)} + 4\sqrt{(4 \times 2)}$
= $3 \times 5\sqrt{2} - 5 \times 4\sqrt{2} + 4 \times 2\sqrt{2}$

$$= 15\sqrt{2} - 20\sqrt{2} + 8\sqrt{2}$$

= $3\sqrt{2}$

4.0 CONCLUSION

The above analysis shows that number system is the foundation of any mathematical analysis. It cuts across all disciplines, it is used daily by every individual in daily life be it at home, office, or business. It is very essential to know the basics of numbers as the means of evaluation of any transaction; this is because numbers help us to measure quantity, price and other variables of life.

5.0 SUMMARY

In this unit, you have been introduced to the meaning and scope of numbers, even though the scope is wide and in-exhaustive, the basic foundational knowledge of numbers will help you cope with the challenges of other courses. The unit therefore examined the basic concepts of numbers as a means of helping you to study other units effectively.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. a) Simplify $\sqrt{45}$ x $\sqrt{27}$
 - b) Evaluate (i) 40 (28)
 - (ii) 48 - 11
 - (iii) 10 - 18
- 2. Write explanatory notes on the followings
 - a) Natural number
 - b) Whole number

7.0 REFERENCES/FURTHER READING

- Bello, S.O (2001). *Basic Mathematics with Applications*. Ilaro: Omolaja Educational Press.
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- Throsby, C.B. & James, D.E. (1972). *Introduction to Quantitative Methods in Economics*. New York: John Wiley and Sons.

UNIT 2 SIMPLE FRACTIONS

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 - 3.1.3 Mixed Numbers
 - 3.2 Addition and Subtraction of Fractions
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 - 3.4 Fractions Involving Bracket
 - 3.5 Application of Fractions
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
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1.0 INTRODUCTION

A fraction is a part of a whole number. A whole number is called an integer, such as 1, 2, 3 - - - 100. A whole number is a combination of pieces of fractions. For example, a thirty centimeter ruler can be cut into six equal parts; each part will be five centimeters long. Each of the pieces is a fraction of the whole ruler. The piece is called one–sixth and can be denoted $^{1}/_{6}$. Equally, each centimeter of the ruler is one–thirtieth ($^{1}/_{30}$). In this fraction one (1) is called the numerator and thirty (30) is called the denominator.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define a fraction
- identify different types of fraction
- work some fractions involving addition, subtraction and multiplication
- work applications involving fractions.

3.0 MAIN CONTENT

3.1 Types of Fractions

There are three basic types of fractions in mathematical analysis. They are proper fractions, improper fractions and mixed numbers.

3.1.1 Proper Fractions

A fraction is classified as proper fraction when the numerator of a fraction is smaller than the denominator. In other words the denominator should be bigger than the numerator. For examples; $\frac{1}{2}$, $\frac{2}{3}$.

3.1.2 Improper Fractions

An improper fraction exists when the numerator of a fraction is greater than the denominator e.g. $\frac{3}{2}$, $\frac{5}{4}$, $\frac{8}{2}$

3.1.3 Mixed Numbers

If a number consists partly of an integer and a fraction, this is called a mixed number e.g. $3 + \frac{1}{10}$, which may also be written as $3 \frac{1}{10}$.

3.2 Addition and Subtraction of Fractions

The addition and subtraction of fractions come in different ways. These are, addition and subtraction of fractions, with the same denominators and addition and subtraction of fractions with different denominators

3.2.1 Addition and Subtraction of Fractions with the same Denominators

Example 1 Addition of fractions with the same denominators

$$2^{2}/_{7} + {}^{3}/_{7}$$

$$= \frac{2+3}{7}$$

$$= {}^{5}/_{7}$$

Example 2 subtraction of fractions with the same denominators

$$\begin{array}{rcrcr}
 & 4y & - & \underline{2y} \\
5 & & 5 \\
 & & 4y & - & 2y \\
\hline
 & & 5 \\
 & & & 5
\end{array}$$

$$= & \underline{2y} \\
5 \\$$

Example 3
$$\frac{3x}{5} - \frac{8y}{5}$$
$$= \frac{3x - 8y}{5}$$

3.2.2 Addition and Subtraction of Fractions with different Denominators

Addition of fractions with different denominators: once the fractions have different denominators, find a common factor as the lowest common multiple (LCM) as the common denominator. The lowest common multiple is the smallest number that can be divided without remainder by all the numbers of the given set of fractions.

Example 4
$${}^{5}/_{6} + {}^{3}/_{8}$$

Find the lowest common factor. This equals 24. It is the lowest number that constitutes the exact multiple of 6 and 8.

Example 5

Simplify the following fractions: $^{7}/_{10}$ - $^{2}/_{15}$

The lowest common factor of 10 and 15 is 30 Therefore we have

$$\frac{21}{30} - \frac{4}{30} = \frac{21 - 4}{30} = \frac{17}{30}$$

Example 6

Simplify the following

$$5y - y = 5y - 2y = 5y - 2y = 3y = y$$

SELF-ASSESSMENT EXERCISE

Simplify the following fractions

a)
$$\frac{2x}{3} - \frac{3y}{5}$$

b) $\frac{5m}{12} - \frac{3n}{8}$
c) $\frac{3y}{4} + \frac{y}{6}$

3.3 Multiplications of Fractions

In the multiplication of fractions, the numerators are multiplied together and the denominators are also multiplied together to form a common whole fraction.

Example 7

Multiply the following fractions:

$$^{4}/_{6}$$
 $x^{8}/_{10} = ^{32}/_{60}$

If the fractions that would be multiplied have numerators and denominators that have common factors, it is more ideal to reduce them through division before the multiplication.

Example 8

Multiply the fraction below:

$$^{4}/_{6} \times 8/_{10}$$

The fractions can be reduced since they have a common factor of 2.

$$\frac{4}{6}$$
 x $\frac{8}{10}$ = $\frac{2}{3}$ x $\frac{4}{5}$ = $\frac{8}{15}$

Example 9

Multiply the fractions below

$$\begin{array}{lll}
 & 14 \\
/_{10} & x & 6 \\
/_{7} & x & 8 \\
 & = 2 \\
/_{5} & x & 2 \\
/_{1} & x & 4 \\
/_{3} & = & 16 \\
/_{15} & = & 1^{1} \\
/_{15}
\end{array}$$

When mixed numbers are given as part of a fraction, they should be all converted into improper fraction before multiplication is carried out.

Example 10

Solve the mixed numbers below $5\frac{1}{2} \times 2^{2}/_{7} \times 5^{-5}/_{33}$.

The $5\frac{1}{2}$ and $2^2/_7$ should be converted to improper fraction. They become $^{11}/_2$ and $^{16}/_7$

Collect the fraction together for multiplication

$$\frac{11}{2} \times \frac{16}{7} \times \frac{5}{33}$$

$$\frac{11}{2} \times \frac{16}{7} \times \frac{5}{33} = \frac{1}{1} \times \frac{8}{7} \times \frac{5}{3} = \frac{40}{21} = \frac{1}{19} \times \frac{1}{21}$$

SELF-ASSESSMENT EXERCISE

Solve the following fractions

$${}^{5}/_{6} + {}^{2}/_{9}$$
, 2). ${}^{7}/_{12} - {}^{3}/_{8}$, 3). $12 \, {}^{1}/_{10} + 5 \, {}^{4}/_{15} - 7 \, {}^{3}/_{5}$, 4.) ${}^{4y}/_{9} \times {}^{3}/_{2}$, 5.) ${}^{5y}/_{6} \times {}^{9}/_{y}$

3.4 Fractions Involving Brackets

Fractions involving brackets are usually mixed equations. This is because they are made up of fractions and integers (whole numbers).

Example 11

Solve the following fractions with bracket

$$\frac{3}{8} (y + 7) + \frac{5}{6} (2y - 3)
= \frac{3}{8} (y + 7) + \frac{5}{6} (2y - 3)
= 3 (y - 7) + 5 (2y - 3)
8 6$$

Find a lower common multiple of 8 and 6 which is 24 the lower common multiple becomes a means of forming a common denominator as follows

$$\frac{9 (y + 7) + 20(2y - 3)}{24}$$
=
$$\frac{9y + 63 + 40y - 60}{24}$$
=
$$\frac{49y + 3}{24}$$

SELF-ASSESSMENT EXERCISE

Solve the following fractions

i.
$$\frac{3}{8}(4x-5) - \frac{5}{12}(3x-5) = \frac{1}{6}$$

ii. $\frac{4}{5}(2y+5) = \frac{2}{3}(2y+7) - \frac{2}{15}$

3.5 Application of Fractions

The application of fractions is an illustration of circumstances in real life where the knowledge of fractions can be used to solve daily problems.

Example 12:

A cyclist made a journey of 152km in a total time of $3^{1}/_{2}$ hours. He went part of the way at an average speed of 40km/h and for the rest of the journey the cyclist averaged 48km/h. How many kilometers did the cyclist cover at 40km/h and 48km/h.

Solution

Assuming the cyclist traveled y kilometers at 40km/h the time taken is y_{40} hours(1)

The remaining part of the journey was (152 - y) kilometers and he traveled this at 48km/hour.

The time taken for this journey = $^{152-y}/_{48}$ hours(2) The total time for the cycling was 3½ hours.

Therefore:
$$y + 152 - y = 3\frac{1}{2}$$

The cyclist went 80km at 40km/hour the rest of the journey can also be determined.

$$(152 - y) \text{ km}$$

 $(152 - 80) \text{ km} = 72 \text{km}$
The cyclist covered 72km at 48km/hor

The cyclist covered 72km at 48km/hour

4.0 CONCLUSION

The above analyses show that fractions are vital in business and daily life applications and should be encouraged. Therefore it is very essential for you to get involved in solving problems relating to fractions as it can be practically applied in your business transactions and daily living.

5.0 SUMMARY

In this unit you have learnt about simple fractions, proper and improper fractions, addition, subtraction and multiplication of fractions. Fractions with bracket were also examined to give you a broad knowledge of the topic. The application of fractions was also examined for you to appreciate the fact that this arithmetic can be applied daily in life and business.

6.0 TUTOR-MARKED ASSIGNMENT

- (a) $y = 2^{1}/_{2}$, find the value of $2y^{2} 3y + 1$ (b) if $y = 2^{1}/_{4}$, find 1. the value of $\frac{2}{3}$ of y
- 2. In a 60km bicycle race a rider calculates that if he can increase his speed by 6km/h., he will cut his time for the distance by 20 minutes. What was his original speed?
- 3. A man bought a certain number of packets of matches at N1.26k. He kept 4 packets for his own use and sold the rest at 3k more per

packet than he paid for them, making a total profit of 14k on the business. How many packets of matches did he buy?

7.0 REFERENCES/FURTHER READING

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UNIT 3 EXPONENTS AND ROOTS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Laws of Exponents and Examples
 - 3.2 Multiplication of Exponents
 - 3.3 Division of Exponents
 - 3.4 Roots of Exponents
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

Exponential functions are commonly used in business and economics in growth theories. Exponential functions are applied to solve optimization equations and problems that use time as part of the choice variable. Therefore, they are used to express functions that grow overtime and the time is measurable through the application of the knowledge exponents and roots.

Exponential functions can also be used to express and find solutions to variables involving compound interest, annuities and sinking fund as it relates to business and economics.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- apply the laws of exponents
- explain the multiplication of exponents
- explain the division of exponent
- explain exponents involving roots.

3.0 MAIN CONTENT

3.1 The Laws of Exponents

The laws of exponents can be expressed by the different use of exponents in different ways such as follows.

1. When multiplications of two exponents are given, the exponents are added.

$$y^m\;y^n=y^{m+n}$$

Example 1:1

$$5^2 \times 5^4 = 5^{2+4} = 5^6 = 15625$$

2. When a number has an exponent and it is multiplied by another exponent then the product of it is the multiplication of the two exponents.

$$(y^m)^n = y^{mn}$$

Example 2:

$$(3^2)^3 = 3^{2x3} = 3^6 = 729$$

3. When an exponent is to be divided by another exponent the result is the subtraction of the exponential numerator from the denominator.

$$\frac{\mathbf{y}^{\mathbf{m}}}{\mathbf{v}^{\mathbf{n}}} = \mathbf{y}^{\mathbf{m}-\mathbf{n}}$$

Example 3:

$$\frac{4^6}{4^2} = 4^{6-2} = 4^4 = 256$$

4. If any variable is raised to a zero exponent, the product of it is one.

$$y^0 = 1$$
. **Example 4** $88^0 = 1$

5. When an exponent is a product of two variables, it is converted to the first variable multiplied by the second variable each raised to the same exponent. $(xy)^n = x^n y^n$

Example 5:

$$(2x5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100.$$

6. When two variables that divide each other are raised to a common exponent, it is converted to the two independent variable raised to the exponent $(x/y)^n = x^n/y^n$ where $y \neq 0$;

Example 6:

Solve the exponent $(2/5)^5 = 2^5/5^5 = {}^{32}/_{243}$

7. A negative exponent is the reciprocal of the number to be determined.

Example:
$$y^{-n} = \frac{1}{y}^{n}$$
, $y \neq 0$ Example 7: solve $4^{-2} = \frac{1}{4^{2}} = \frac{1}{16}$

SELF- ASSESSMENT EXERCISE

Discuss and illustrate the laws of exponents.

3.2 **Multiplication of Exponents**

When variables are raised to a given exponent that should be multiplied, the result is the sum of the given exponents.

Example 8 Solve the equation
$$x^4 \times x^7 = x^{4+7} = x^{11}$$

Example 9 Solve the equation
$$4^3 \times 4^2 = 4^{3+2} = 4^5 = 1024$$

Example 10 Multiply
$$3c^3e^2$$
 by $2c^2e^2 = 6c^{3+2}e^{2+2} = 6c^5e^4$

SELF-ASSESSMENT EXERCISE

Given the following variables, find the solution with respect to the exponents.

a)
$$y^4 \times y^2$$

b) $6^3 \times 6^2$

b)
$$6^3 \times 6^2$$

Division of Exponents 3.3

Example 11: Simplify the following, divide $-12x^4y^3z^2$ by $-4x^3y$

$$= \frac{12x^4y^3z^2}{-4x^3y}$$

$$= \frac{12x \times x \times yyyzz}{4xxxy}$$

$$\frac{3xy^2z^2}{}$$

Example 12: Divide the following exponents: y^5 by y^2

$$\frac{y^5}{y^2} = \underbrace{\begin{array}{ccc} y & y & y & y \\ y & y & y \\ & & & y^3 \end{array}}_{}$$

Example 13: Divide the exponent 64 by 6^2 = $\frac{6^4}{6^2} = 6^{4-2} = 6^2 = 36$

$$= \frac{6^4}{6^2} = 6^{4-2} = 6^2 = 36$$

3.4 Exponents and Roots

Sometimes exponents are expressed as roots or a product of some roots. This can be solved using the same laws of exponents.

Example 15: Simplify
$$3\sqrt{y^{21}}$$

$$= y^{21 \div 3}$$

$$= y^7$$

Example 16: Solve
$$3 \sqrt{(-8y^{15}n^3)}$$

= $-2y^{15 \div 3} n^{3 \div 3}$
= $-2y^5 n^1$
= $-2y^5 n$
Notice that $3\sqrt{8} = 2$ or $(-2)^3 = 8$

Example 17: Simplify
$$10 \times {}^{2}y + 6 \times {}^{2}y^{2} - 8 \times {}^{2}y^{2}$$

$$= \underbrace{10x^{2}y}_{2xy} \times \underbrace{6 \times y^{2}}_{2xy} = \underbrace{8x^{2}y^{2}}_{2xy}$$

$$= 5x + 3y - 4xy$$

SELF-ASSESSMENT EXERCISE

Simplify: i.
$$4\sqrt{(81x^8y^4)}$$
 iii. $3\sqrt{(27y^{12})}$ iv. $3\sqrt{(125R^6)}$

3.5 Fractional Exponents

There are circumstances in which the exponents can be expressed in fractions. The solution follows the same rules of working exponents.

Example 18:
$$y^{3/4} = y^{1/4} \times y^{1/4} \times y^{1/4} = (y^{1/4})^3 = (4\sqrt{y})^3$$

$$16^{3/4} = {}^4\sqrt{(16^3)} = {}^4\sqrt{(2^4 \times 2^4 \times 2^4)} = {}^4\sqrt{2^{12}} = 2^3$$
Alternatively $16^{1/4} = 4\sqrt{16} = 2$
 $16^{3/4} = 2^3$

Example 19:Simplify
$$10^{3/4}$$

= $4\sqrt{10^{3}} = 4\sqrt{1000} = \sqrt{31.62} = 5.623$

SELF-ASSESSMENT EXERCISE

Simplify a) $16^{1/2}$ b) $8^{2/3}$ c) $81^{3/4}$ d) $100^{2/3}$

4.0 CONCLUSION

Exponential functions are applied both in business arithmetic, economics and other social sciences. A good knowledge of exponents and roots can assist tremendously in enhancing your knowledge.

5.0 SUMMARY

In this unit, you have learnt about exponents and roots. You have been introduced to the concepts of exponents and roots by using simple symbols of multiplication, addition and division to assist you. Ample examples were given to drive home the points explained in the unit.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. Find x if $2^x \times 4^{2x+3} \times 8^{x-1} = 16$.
- 2. Find the value of x if $3^{2x+1} 28(3^x) + 9 = 0$

7.0 REFERENCES/FURTHER READING

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UNIT 4 RATIOS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 The Concept of Ratio
 - 3.2 Increase and Decrease in Ratio
 - 3.3 Comparison of Ratio
 - 3.4 Workings of Ratio and Applications
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

A ratio shows the number of times one quantity or unit contains another. It is used to show the relationship between two amounts. Here, the comparison is made in the form of a ratio that is the fraction which the first quantity is of the second. Suppose a company has 150 men and 200 women, then the number of men is $\frac{3}{4}$ of the number of women and we say that the ratio of the number of men to the number of women is 3 to 4, written 3:4 and this ratio can be represented by the fraction $\frac{3}{4}$.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the term ratio
- discuss the increase and decrease in ratio
- explain and discuss comparison of ratio.

3.0 MAIN CONTENT

3.1 Explanation of the Term and Concept of Ratio

Ratio is a relationship between two amounts of quantity in which one relates to another. Ratios should be expressed as simply as possible, just as the fraction $^{8}/_{36}$ can be reduced to $^{2}/_{9}$, so the ratio 8:36 is equivalent to 2:9. Therefore, a ratio is unaltered if the two numbers or quantities of the ratio are both multiplied, or both divided by the same number. Example, the ratio $^{5}/_{6}$: $^{3}/_{4}$ equals the ratio $^{5}/_{6}$ x 12 : $^{3}/_{4}$ x 12 that is 10:9.

When we want to express the prices of two books x and y in ratio, e.g \times 720 and \times 960 respectively, It is done as follows.

$$\frac{\text{Price of x}}{\text{Price of y}} = \frac{720}{960} = \frac{4/3}{960}$$

$$\frac{\text{Similarly}}{\text{Price of y}} = \frac{960}{720} = \frac{4}{3}$$

We write the price of x : price of y as equal to 3:4 and the price of y : price of x as 4:3. Conversely, the statement that the ratio of the price of x to the price of y is 3:4 means that the price of x is $\frac{3}{4}$ of the price of y and that the price of y is $\frac{4}{3}$ of the price of x.

SELF- ASSESSMENT EXERCISE

Explain the term ratio and illustrate it with examples.

3.2 Increase and Decrease in Ratios

Ratios can depict an increase and decrease in the occurrence of a given event, or numbers. If the daily price of a ticket is raised from $\frac{N}{4}$ 60 to $\frac{N}{4}$ 80, the ratio of the new price to the old price of ticket equals 80:60 = 4:3 we can say that the price of the ticket has increased in the ratio 4:3. In other words, the new ticket price is $\frac{4}{3}$ times the old ticket price. If the daily price per ticket for entering a cinema is lowered from $\frac{N}{4}$ 60 to $\frac{N}{4}$ 8, the ratio of the new ticket to the old ticket price would be $\frac{48:60}{4:5}$, and we say that the ticket price has been reduced in the ratio of 4:5. In other words the new ticket price is $\frac{4}{5}$ times the old price.

The fraction $\frac{4}{5}$ by which the old ticket price $\frac{1}{8}$ 60 must be multiplied to give the new ticket price of $\frac{1}{8}$ 48 is called a multiplying factor.

 $\underline{\underline{New \ quantity}} = \underline{\underline{Multiplying}} \ factor$

Old quantity

The multiplying factor is less than one if the new quantity is less than the old quantity; it is greater than one if the new quantity is greater than the old quantity.

Example 1

Umenemi Nig Ltd water wants to increase its water rate of 56k in the ratio 10:7. Determine the new water rate.

Increased value = $56k \times {}^{10}/_{7}$

$$= \frac{56 \times 10}{7}$$
$$= 80k$$

Example 2

Okewa bread wants to reduce the time taken for baking of 2 hours in the ratio of 5:6. What is the decreased time for baking?

Decreased time =
$$2 \text{hrs x}^{5}/_{6}$$

= $\frac{2 \times 5}{6}$
= $\frac{5}{_{3}}$ hours
= 1 hour 40 minutes

Example 3

In what ratio should $\frac{100}{75}$ be increased to become $\frac{100}{75}$ The ratio $\frac{100}{75}$ = $\frac{10$

3.3 Comparison of Ratios

We have stated that a ratio is a relationship, a ratio may be expressed in the form n: 1, where n is a whole number, a fraction, or a decimal calculated to any required degree of accuracy. This is particularly important when comparing ratios.

Example 4:

Express the ratio of 4.10:1.90 in the form n:1

$$\frac{4.10}{1.90} = \frac{4.1}{1.9}$$

$$= \frac{41 \div 19}{1}$$
 dividing the numerator and denominator by 19
$$= \frac{2.16}{1}$$

:- The ratio is 2.16:1

Example 5:

Find which ratio is greater 7:13 or 8:15 $^{7}/_{13} = 0.538$ therefore 7:13 = 0.538:1 $^{8}/_{15} = 0.533$ therefore 8:15 = 0.533:1

The first ratio is greater than the second. The first gives the value 0.538 while the second has the value 0.533.

SELF-ASSESSMENT EXERCISE

Find which ratio is greater from the following 9:16 or 7:1

3.4 Applications of Ratios

Ratio system is used by planners, geographers and geographical information system and other forms of surveys.

For map and plans, the ratio is usually in the form of 1:n. For example if the scale on a map is 5cm to the kilometer, 5cm on the map represents one kilometre on the ground based on survey specifications.

5cm: 1km = 5cm: 100000cm = 1:20000

Therefore the scale of the map is 1:20000

The fraction $^{1}/_{20000}$ is called the representative fraction. Note that a scale of 1:16 is greater than a scale of 1:17 since $^{1}/_{16}$ is greater than $^{1}/_{17}$.

Example 6

Express the ratio 8:13 in the form 1: n $^{8}/_{13} = ^{1}/_{(13 \div 8)}$ dividing numerator and denominator y 8 $= ^{1}/_{1.625}$ \therefore The ratio is 1:1.625.

Example 7

If 5 people dig the foundation of a house in 14 days, how long would 7 people take to dig the foundation?

Solution

Since the number of men had increased, it will take them less days to dig the foundation. This can be expressed in a ratio.

The number of men increased in ratio 7:5

Therefore the time taken is decreased in the ratio 5:7

What is to be calculated is the time.

5 men = 14 days 7 men = 14 x $^{5}/_{7}$ days = 10 days

Example 8:

In a market, $2\frac{1}{4}$ kg. of coffee cost $\frac{1}{1}$ 1.17. What quantity of coffee can be bought for N1.95 in the market?

Solution It is given that $\aleph 1.17$ is the cost of $2\frac{1}{4}$ kgand $\aleph 1.19$ is the cost of

SELF-ASSESSMENT EXERCISE

- i. If 10 people dig the foundation of a house in 28 days, how long would 16 people take to dig the foundation?
- ii. In a market 4¼ kg. of coffee cost № 118 what quantity of coffee can be bought for № 295 in the market?

4.0 CONCLUSION

In this unit, you have learnt about analysis of ratio which shows that it is important in life that we need to make comparison of events. These range from daily comparison of sales, cost, work and output to measurement of geographical area, and presentation of such data for human use. It is therefore important for you to learn about ratio and practise the applications of ratios as well.

5.0 SUMMARY

The unit has shed some lights on the meaning of ratio, comparison of ratios, increase and decrease of ratios, workings and application of ratios. The ratio system is important for every practising manager.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. Find the ratio x:y if $6x^2 = 7xy + 20y^2$
- 2. (a) A man takes 18 minutes for a journey if he travels at 20km. per hour. How long will the journey take if he travels at 24km. per hour?
 - (b) A car takes 50 minutes for a journey if it runs at 72km/h. At what rate must it run to do a journey of 40 minutes?

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UNIT 5 ANALYSIS OF VARIATION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Direct Variation
 - 3.2 Inverse Variation
 - 3.3 Joint Variation
 - 3.4 Partial Variation
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

Variation is a mathematical method of finding the rate of change in quantities, volumes, speed or any other event or group of events that depend on each other. Variation could be direct, inverse, joint or partial.

Variation as a unit embraces all aspects of daily life activities. Activities depend on one another. For example, the ability to work depends on our health, energy and other utilities, the ability to drive depends on expertise, or experience, the type of vehicle and the nature of the road and the degree of concentration. In this unit, you will be introduced to some of these interrelationships and their applications.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the term variation
- discuss direct variation
- discuss inverse variation
- discuss joint variation.

3.0 MAIN CONTENT

3.1 Direct Variation

If two variables (quantities x and y) are so related that the ratio of simultaneous values of x and y is constant then, either quantity varies directly as the other quantity.

Steps in solving variation problems are as follows:

- a. Change the given statement into a mathematical expression involving α , where α is the proportionality symbols.
- b. Replace α by k in the new mathematical expression, where k is a constant.
- c. Find the value k using the initial values and substitute into the equation in step b
- d. Solve the problem using equation in step b free of k.

Example 1

If y varies directly as the square root of x and y = 12 when x = 4. Find y when x = 9.

Y
$$\alpha \sqrt{X}$$
(1)
Y = k \sqrt{X} (2)
When y = 12, x = 4 substitute into equation (2)
12 = K $\sqrt{4}$
12 = 2K
Find K by divide through by 2
 $^{12}/_2 = ^{2k}/_2$
K = 6
Substitute for k in equation 2 where x = 9
Y = 6 \sqrt{X}
Y = 6 $\sqrt{9}$
= 6 x 3
Y = 18.

SELF-ASSESSMENT EXERCISE

If W varies directly as the square root of V and W=24 when V=8 find W when V=18.

3.2 Inverse Variation

When two variables x and y are related in such a way that the quotient obtained on dividing x by the corresponding value of $^{1}/_{y}$ is a constant, then, x is said to vary inversely as y. Therefore if y varies inversely as x, y varies directly as $^{1}/_{x}$.

Example 2:

The electrical resistance of a wire varies inversely as the square of its radius. Given that the resistance is 0.4 ohms when the radius is 0.3cm, find the resistance when radius is 0.45cm.

Let R be the resistance in ohms and r the radius in cm.

Therefore R
$$\alpha^{1}/r^{2}$$
 (1)
R = $^{K}/r^{2}$ (2) where r is a constant

When R = 0.4, r= 0.3 substitute into equation (2)

$$0.4 = \frac{k}{(0.3)^2}$$

 $K = (0.4) (0.3)^2 = 0.036$
 $R = \frac{0.036}{r^2}$
When r = 0.45 substitute into equation (2)
 $R = \frac{0.036}{(0.45)^2}$
 $= 0.18$

Example 3:

If y is inversely proportional to Z^2 and if y = 4 when Z = 3. (i) Find the value of y when Z = 4 and the positive value of Z in terms of y $Y = {}^{K}/{}_{Z}^{2}$ when K is a constant

Since
$$y = 4$$
, when $Z = 3$.

$$K = 4 \times 3^2 = 36$$

$$y = \frac{36}{Z^2}$$

When
$$Z = 4$$
, $y = \frac{26}{16} = 2\frac{1}{4}$

Since
$$Z^2 y = {}^{36}/_y$$

 $Z^2 = {}^{36}/_y$, :- $Z = {}^6 / \sqrt{y}$

SELF-ASSESSMENT EXERCISE

If y varies inversely as \sqrt{x} and if y = 5 when x = 16, find y if x = 100 and find x if y = 60. Find also y in terms of x.

3.3 Joint Variation

When one quantity varies as the product of two or more quantities, then it is called joint variation.

Example 4:

If v values directly as the square of x and inversely as y and if v = 18 when x = 3 and y = 4. Find v when x = 5 and y = 2

$$V \alpha x^2/y......$$
 (1)

: - V =
$$(Kx^2)/y$$
(2) where K is a constant

When v = 18, x = 3 and y = 4 then substitute into equation (2)

$$18 = \underline{K(3)^2}$$

$$18 = 9k$$

$$K = \frac{18 \times 4}{9} = 8$$

Therefore

$$V = \frac{8x^2}{Y}$$

When $x = 5$, $y = 2$
 $V = \frac{8 \times (5)^2}{2} = \frac{8 \times 25}{2} = 4 \times 25 = 100$
 $V = 100$

SELF-ASSESSMENT EXERCISE

- i. If y varies directly as the square of x and inversely as w and if y = 36 when x = 6 and y = 8 find y when x = 10 and y = 4
- ii. If w varies jointly as L and the square of r. find the percentage change in w if L increases by 20% and r increases by 50%. If w = 15 when h = 3 and $r = 2\frac{1}{2}$, find w when h=1 and r = 10; find also w terms of h, r.

3.4 Partial Variation

This is a situation where a function varies partly as the sum or difference of two quantities. For partial variation, there are at least two constants. These constants have to be found first before solving the question. For the computation of partial variation the procedures are slightly modified as follows:

- a. Change the statement to a mathematical expression
- b. The values given together with the mathematical expression formulate two equations, with two unknowns
- c. Solve the two equations in step (b) simultaneously to obtain the values of the constants.
- d. The problem can now be solved with the mathematical expression free of the constants.

Example 5:

Given that y is the sum of two quantities, one of which varies as x and the other which varies inversely as x. If y = 20 when x = 1 and y = 12 when x = 3, find the values of y when x = 6.

Then $a \alpha x$, a = cx where c is a constant

Also b α 1/c

Then b = n/x, where n is a constant (n = c)

Substituting for a and b in equation (1)

$$Y = cx + n/x \dots (2)$$

Substitute when y = 20, x = 1 into equation (2)

$$20 = c + n \dots (3)$$

Substitute when y = 12, x = 3 into equation (2)

$$12 = 3c + n/3$$

```
36 = 9c + n....(4)
Now solve equation (3) and (4) simultaneously when c = 2 and n = 18
substitute into (2)
Y = 2x + 18/x .....(5)
Therefore when x = 6 substitute for x in equation (5)
Y = 2(6) + 18/6
= 12 + 3
= 15
When x 6, y = 15.
```

Example 6:

The volume of a given mass of gas varies directly as the absolute temperature and inversely as the pressure. At absolute temperature of 360° and at pressure of 736mm the volume is 450cm³; find a general formula and find the volume at absolute temperature 312⁰ and pressure 960mm.

Solution

```
If the volume is vcm<sup>3</sup> at absolute temperature T<sup>0</sup> and pressure Pmm
V \alpha T/p
V = K \times T/p
Where K is constant, when T = 360 and P = 736, V = 450
450 = K \times 360/736
:- K = 450 \times 736
            360
       = 920
V = 920T
        P
When T = 312 and P = 960
V = 920 \times 312
         960
    = 299.
```

SELF-ASSESSMENT EXERCISE

If Z varies directly as the square of x and inversely as the square root of y, find the percentage change in Z if x increases by 20% and y decreases by 19%. If Z = 3 when x = 6 and y = 16, find Z when x = 12 and y = 25; find also Z in terms of x and y.

4.0 **CONCLUSION**

The volume is 299cm³

In this unit, you have learnt that variation has a wide range of usage and applications. Attempts had been made within the time limit and scope to present what can assist you in the analysis of mathematics in other levels of your study.

5.0 SUMMARY

The unit has examined the combined theory of variation and practice and applications. The unit examined direct, inverse, joint and partial variations to drive home the concept of variation. Examples used in the exercises were such that can assist you in your independent studies.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. If y varies directly as Z and y = 10 when Z = 6, find the value of Z when y = 12.5
- 2. (a) R α m and R = 6, when m = 16. Find the law connecting R and M. find R when m = $6\frac{1}{4}$ and m when R = 15.
 - (b) Given that y varies directly as X^2 . How is the value of y affected if the value of x decrease by 20%?

7.0 REFERENCES/FURTHER READING

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MODULE 2 INTRODUCTION TO EQUATION SYSTEM

Unit 1	Linear Equation
Unit 2	Simultaneous Linear Equation
Unit 3	Quadratic Equation
Unit 4	Analysis of Progressions

UNIT 1 LINEAR EQUATION

CONTENTS

- 1.0 Introduction
- 2.0 Objective
- 3.0 Main Content
 - 3.1 Addition and Subtraction of Linear Equation
 - 3.2 Multiplication of Linear Equation
 - 3.3 Division of Linear Equation
 - 3.4 Applications of Linear Equation
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

A linear equation is a mathematical statement or an expression that has an unknown variable. The unknown variable is raised to the power of one. A linear equation usually may have a constant that connects the equation with the unknown. The equation is usually connected by an equality (=) sign.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- solve problems of linear equation involving addition and subtraction
- solve problems of linear equation using multiplication method
- solve linear equation using division
- solve application problems involving linear equation.

3.0 MAIN CONTENT

3.1 Addition and Subtraction of Linear Equation

It was earlier stated that when two things are equal, it means they can be estimated quantitatively and the process of estimation is called equation.

Example 1:

Find the value of the unknown variable

$$14y = 28$$

Find the value of y that is unknown

$$14y = 28$$

Divide both sides by 14 and it becomes

$$\begin{array}{ccc}
\underline{14y} & = & \underline{28} \\
14 & & 14
\end{array}$$

$$Y = \underline{2}$$

Example 2:

Find the value of x from the following

$$3x + 2 = 2x + 10$$

Collect like terms

$$3x - 2x = 10 - 2$$

$$X = 8$$

Example 3:

Find the value of y from the following

$$30x + 10 + 2x = 15x + x + 42$$

Collect like terms

$$30x + 2x - 15x - x = 42 - 10$$

$$32x - 16x = 32$$

$$16x = 32$$

Divide through by 16 we have

$$\underline{16x} = \underline{32}$$

$$X = \underline{2}$$

SELF-ASSESSMENT EXERCISE

Find the value of the unknown in the following.

i.
$$8 - 19 = 5 - 3y$$

ii.
$$4 - 3x = -7x + 8$$

iii.
$$6x + 7 - 5x = 19 - 2x - 3$$

3.2 Multiplication of Linear Equation

In the multiplication of linear equation, the necessary expansion of the equation should first be carried out, then the value of the unknown can be determined.

Example 4:

Find the value of the unknown from the equation below.

$$3(x + 3) = 2(0.5x + 7)$$

First clear the bracket through multiplication.

$$3(x + 3) = 2(0.5x + 7)$$

$$3x + 9 = x + 14$$

Collect like terms 3x - x = 14 - 9

$$2x = 5$$

Divide through by 2 to find the value of x

$$\frac{2x}{2} = \frac{5}{2}$$

$$X = \underline{2.5}$$

Example 5:

Find the value of y in the following equation.

$$y(10-2) = 80$$

$$10y - 2y = 80$$

$$-8y = 80$$

$$:- \underline{8y} = \underline{80}$$

$$8 8$$
$$Y = 10.$$

Example 6:

Solve the equation
$$(5)y = 2y + 7$$

$$(5)y = 2y + 7$$

Subtract 2y form both side

$$5y - 2y = 2y + 7 - 2y$$

$$-3y = 7$$

Divide both sides by 3 to find the value of y

$$3y = 7$$

$$Y = 2^{1}/_{3}$$

Example 7:

Solve the equation 22 = (7)y - 6

Add 6 to both sides of the equation

$$22 + 6 = 7y - 6 + 6$$

$$28 = 7y$$

divide both sides of the equation by 7 to find the value of y 28/7 = 7y/7, y = 4

SELF-ASSESSMENT EXERCISE

Solve the following equations

- i. (3)x 2 = 10
- ii. 10(x-2) = 2(x+1)
- iii. x(15 + 4) = 5(x + 2)

3.3 Division of Linear Equation

In the division of linear equation, an understanding of the process of multiplication of linear equation is needed. |The understanding of the multiplication process helps in simplifying the equation to determine the value of the unknown variable.

Example 8: Solve the equation

$$\frac{2y+5}{y} = \frac{2y}{y-3}$$

Cross multiply the equation to clear the division

$$(2y + 5) (y - 3) = (2y)y$$

Open the bracket and multiply the variables

$$2y^2 + 5y - 6y - 15 = 2y^2$$

Collect the like terms

$$2y^2 - 2y^2 + 5y - 6y = 15$$

$$5y - 6y = 15$$

$$-y = 15$$

Example 9;

Solve the equation

$$\frac{10x+4}{2} = \frac{2x}{4}$$

Cross multiply

$$\frac{10x+4}{2} = \frac{2x}{4}$$

Cross multiply

$$\frac{10x+4}{2} = \frac{2x}{4}$$

$$(10x + 4)4 = 2(2x)$$

$$40x + 16 = 4x$$

Collect like terms 40x - 4 = -16

$$36x = -16$$

$$X = \frac{-16}{36}$$

Example 10

. Solve the equation
$$\frac{6x + 14}{x} = \frac{14x}{x-8}$$

$$\frac{6x + 14}{x} = \frac{4x}{x - 8}$$

Cross multiply the equation

$$(6x + 14)(x - 8) = x(4x)$$

$$6x^2 - 14x - 48x - 112 = 4x^2$$

Collect like terms

$$6x^2 - 4x^2 + 14x - 48x - 112 = 0$$

$$2x^2 - 34x - 112 = 0$$

Use the formula to solve the equation and find x

$$-\underline{b\pm} \qquad \frac{\sqrt{(b^2 - 4ac)}}{2a}$$

Where a=2, b=-34, c=-12

Substitute into the formula

SELF-ASSESSMENT EXERCISE

or

i. Solve equation

$$\frac{12y + 28}{y} = \frac{8}{16-y}$$

ii. Solve the equation
$$\frac{4x+5}{2} = \frac{4x}{8}$$

3.4 **Applications of Linear Equation**

The equations that have been solved were necessary only to find the number represented by some letters. This section will show how practical problems that involve linear equation can be solved. In each case a letter is introduced to stand for the unknown variable to be calculated.

Example 11:

Emma and Kehinde are to share 454 such that Kehinde has 48 less than Emma. Find the share of each person.

Let's denote Emma's share by x

Kehinde has $\frac{1}{8}$ less than Emma = -8

They share a total of N54

$$x - x + x - 8 = 54$$

Collect like terms x + x = 54 + 8

$$2x = 62$$

Divide through by 2

$$\frac{2x}{2} = \frac{62}{2} \qquad x = 31.$$

Emma's share is N31

Kehinde's share =
$$x - 8$$

= $\frac{N}{2}$ 23.

$$31 - 8$$

Kehinde has N23.

Example 12:

Kufe drove for 3 hours at certain speed and then doubled that speed for the next 2 hours. If Kufe drove the car covering 63kms. altogether, find the speed for the first 3 hours.

Let the speed that he started with = x km/h

Then his speed later on was 2x km/h

Therefore in the first three hours he went 3x km.

And in the next 2 hours he went $2 \times 2 \times km = 4 \times km$

$$3x + 4x = 63$$

$$7x = 63$$

Divide through by 7 to find x

$$\frac{7x}{7} = \frac{63}{7}$$
 $x = 9$

He started at 9km/h

In 3 hours at 9km/h he went (9x3) = 27km

In 2 hours at 18km/h he went (2x18) = 36km

SELF-ASSESSMENT EXERCISE

Emeka cycled for 6 hours at a certain speed and then doubled that speed for the next 2 hours. If the total distance covered was 126kms. altogether (1) find the speed for the first three hours (2) find the distance covered for the period he doubled his speed.

4.0 CONCLUSION

The analysis in this unit demonstrates the fact that linear equation is important in business and managerial decisions. Linear equation can be used to solve problems relating to management practice in companies and even private business establishment. A knowledge of linear equation help to increase the practical application of quantitative reasoning in workplace.

5.0 SUMMARY

In this unit you have learnt the meaning and the application of linear equation. You are now conversant with the addition, subtraction, multiplication and division of linear equations. This is to broaden the scope of your understanding. The applications of linear equations were also treated so that you will not think linear equation is an abstract area of study in Mathematics.

6.0 TUTOR- MARKED ASSIGNMENT

- 1. Solve the equation (i) 4 3x = 17x + 8
 - a) 7 = 9 5y + 8
 - b) $\frac{4}{3} = \frac{x-2}{(x+4)}$
- 2. (a) Paul and Peter received an award of №21,000 as a reward for their excellent performances with a condition that Peter will receive №3,000.00 more than Paul. Determine the amount Peter and Paul will receive.
 - (b) A certain number is multiplied by 8 and then 28 is added, if the result is 100. Find the original number.

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UNIT 2 SIMULTANEOUS LINEAR EQUATION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Solution by Substitution
 - 3.2 Solution by Addition
 - 3.3 Solution by Subtraction
 - 3.4 Application of Simultaneous Equations
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

A simultaneous linear equation is a set of equations with more than one unknown variables, however the number of the unknown variables are usually as many as the set of equation.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define a simultaneous equation
- find the solution to any simultaneous equation by substitution
- solve simultaneous equation by addition
- solve simultaneous equation problem using elimination by subtraction
- work practical problems involving simultaneous equation.

3.0 MAIN CONTENT

3.1 Solution by Substitution

This is a method of finding solution to simultaneous equations where one of the equations is rearranged such that one of the unknown is made the unit and becomes the subject of the equation where it is substituted into the remaining equation, this helps provide a solution to one of the unknowns.

Example 1:

The demand for bread in market x and y is given as follows.

$$3x - 4y = 19$$
(1)
 $x - 2y = 5$ (2)

Determine the value of x and y by substitution.

Make x the subject in equation (2)

$$x = 5 + 2y \dots (3)$$

Substitute the value of x into equation (1)

$$3x - 4y = 19$$

$$3(5+2y) - 4y = 19$$

$$15 + 6y - 4y = 19 \dots (4)$$

Rearrange equation (4) and collect like terms

$$6y - 4y = 19 - 15$$

$$2y = 4$$

$$y = {}^{4}/_{2}$$
.

Substitute the solution of y into equation (3) to determine the value of x.

$$X = 5 + 2y$$

$$X = 5+2(2)$$

$$= 5 + 4$$

Example 2:

Solve the simultaneous equations

$$3w + 2x = 21 - - - (1)$$

$$2w + 5x = 3 - - (2)$$

Using equation (1) solve for x

$$2x = 21 - 3w$$

$$x = \frac{21 - 3w}{2}$$

Substituting equation (3) into equation (2), we have

$$2w + 5(21 - 3w) = 3 - - (4)$$

$$:.4w + 105 - 15w = 6$$

$$4w - 15w = 6 - 105$$

$$-11w = -99$$

$$W = \underline{99}$$

$$= 9$$

Substitute for w into equation (3)

$$X = \frac{21 - 3w}{2}$$

$$= \frac{21 - 3(9)}{2}$$

$$= \frac{21 - 27}{2}$$

$$= -\frac{6}{2} = -3$$

SELF-ASSESSMENT EXERCISE

i. Solve the following simultaneous equations

$$2x - 5y = -3$$

 $3x + 4y = 1$

ii. Solve the simultaneous equations

$$x - 2y = 27$$
$$7x + y = 9$$

3.2 Solution by Elimination using Addition

When the method by substitution involves awkward fractions, it is easier to use the method of elimination by addition or subtraction.

Example 3:

Solve the simultaneous equations

$$3x - 2y = 11 - - - (1)$$

 $5x + 2y = 29 - - - (2)$

Elimination by addition involves adding equation (1) and equation (2) together. When this is done the term y will disappear leaving only x

$$3x - 2y = 11$$

 $5x + 2y = 29$
 $8x + 0 = 40$ - --- (3)

What is left from the equation after elimination by addition is 8x = 40 the value of the unknown variable x can now be determined.

$$x = \underline{40}$$

$$8$$

$$= \underline{5}$$

Substitute x = 5 into equation (1) so that the y unknown can be calculated this gives: 3x - 2y = 11

$$3(5) - 2y = 11$$

 $15 - 2y = 11$
 $-2y = 11 - 15$
 $-2y = -4$
 $y = -4$
 -2
 $= 2$

SELF-ASSESSMENT EXERCISE

Solve the following simultaneous equation by elimination using addition.

i.
$$x + y = 11$$

 $x - y = 5$

ii. Solve the simultaneous equation by addition

$$x - 4y = 2$$
$$x + 4y = 28$$

3.3 Solution by Elimination using Subtraction

This method involves determining the value of the unknown in a simultaneous equation by subtracting one equation from the other, then determine the unknown variables.

Example 4:

Solve the simultaneous equation by elimination using subtraction.

$$2x + 5y = 28 - - - (1)$$

 $2x + 3y = 3 - - - (2)$

When equation (1) is subtracted from equation (2) the term x will become zero and therefore disappears from the equation system.

$$2x + 5y = 28 - (1)$$

$$2x + 3y = 3 - (2)$$

$$0 \quad 2y = 26$$

$$2y = 26$$

$$y = 26$$

$$2$$

$$= 13$$

Substitute y = 13 in equation (2) to determine the value of x, then we have

$$2x + 3y = 3$$

 $2x + 3 (13) = 3$
 $2x + 39 = 3$

Collect like terms 2x = 3 - 39

$$2x = -36$$

$$X = -\frac{36}{2}$$

$$= -18$$

The process of getting rid of one of the unknown variable is known as elimination. It does not matter which unknown is eliminated, the student should always start with the variable that is easy.

SELF-ASSESSMENT EXERCISE

Solve the following simultaneous linear equation

i.
$$2x + 3b = 6$$
 ii. $3a - b = 11$
 $x + 2b = 6$ $2a - 3b = 5$

3.4 Application of Simultaneous Equations

This involves solving problems that we commonly encounter in daily interaction, sometimes it may be in the business transactions and other activities.

Example 5:

In a market survey within Jos, it was discovered that within Ahmadu Bello Way, 6 exercise books and 12 biros cost ¥144. However at Rayfield, 8 exercise books and 10 biros cost ¥132. Determine the price of a biro and an exercise book.

Solution

Let exercise book be represented by x and biro by y we then have: 6x + 12y = 144 - - (1)8x + 10y = 132 - - (2)

Determine the value of exercise book and a biro by multiplying equation (1) by 8 and equation (2) by 6 to bring x variable to the same unit

$$6x + 12y = 144 - - (1) \times 8$$

 $8x + 10y = 132 - - (2) \times 6$
 $48x + 96y = 1152 - - (3)$
 $48x + 60y = 792 - - (4)$

Subtract equation (4) from equation (3)

$$48x + 96y = 1152$$

$$48x + 60y = 792$$

$$0 \quad 36y = 360$$

$$36y = 360 - - - (5)$$

$$y = \frac{360}{36}$$

$$= 10$$

Put the value of y into equation (1)

$$6x + 12y = 144$$

 $6x + 12(10) = 144$
 $6x + 120 = 144$

Collect like terms

$$6x = 144 - 120$$

$$6x = 24$$

$$X = \underline{24}$$

$$6$$

$$= \underline{4}$$

Example 6:

6 years ago Edeh was 3 times as old as Ebo. Their combined age is 24. Determine the age of Edeh and Ebo

Solution

$$x + y = 24 - (1)$$

$$x - 6 = 3(y-6)$$

$$x - 6 = 3y - 18$$

$$x - 3y = -18 + 6$$

$$x - 3y = -12 - (2)$$

The simultaneous equations will be

$$x + y = 24 - (1)$$

 $x - 3y = -12 - (2)$

From equation (1) make x the subject

$$x = 24 - y - (3)$$

Substitute the value of x that is in equation (3) into equation (2)

$$x - 3y = -12$$

$$24 - y - 3y = -12$$

$$24 + 12 = y + 3y$$

$$36 = 4y$$

$$Y = 36$$

$$4$$

$$= 9$$

Substitute the value of y into equation (3)

$$X = 24 - y$$
$$x = 24 - 9$$
$$x = 15$$

Edeh is 15 years while Ebo was 9 years.

4.0 CONCLUSION

In this unit you have learnt that simultaneous linear equation is very vital in business practice and daily interactions. A good knowledge of simultaneous equation will help you solve many common problems.

5.0 SUMMARY

In this unit, you are now conversant with the meaning and scope of simultaneous linear equation. The methods of finding solutions to simultaneous equations examined are elimination by substitution, addition and subtraction. You have also learnt how to solve practical problems on simultaneous equations.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. Solve the following simultaneous equations.
 - a) 6x 5y = 273x + 4y = 16
- b) 3y + 2z = 125y - 3z = 1
- 2. A certain number is formed of two digits; its value equals four times the sum of its digits. If 27 is added to it, the sum is the number obtained by interchanging the digits. What is the number?

7.0 REFERENCES/FURTHER READING

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UNIT 3 QUADRATIC EQUATION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Solution by Factorization
 - 3.2 Solution by Completing the Square
 - 3.3 Solution by Formula
 - 3.4 Solution by Graphical Analysis
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

A quadratic equation is an equation of second degree, that is an equation in which 2 is the highest power of the letter in the equation. There are different methods of determining the solution to quadratic equations. Some of the methods include factorization, completing the square, solution by formula and solution by graphical methods. You are required to study the methods carefully so as to have adequate exposure in quantitative reasoning.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- solve the quadratic equation by factorization
- solve quadratic equation by completing the square
- solve quadratic equation by formula
- solve quadratic equation by graphical method.

3.0 MAIN CONTENT

3.1 Solution by Factors

The method of finding solution to quadratic equation by factors requires looking for appropriate factors for the unknown and the integers within the equation.

Example 1:

$$x^2 - 25 = 0$$

The factors of x^2 and 25 are x and 5 :- $x^2 - 25$;

$$(x + 5) (x - 5) = 0$$

:- either
$$x + 5 = 0$$
 or $x - 5 = 0$

$$x = -5 \text{ or } 5$$

$$x^2 = 25$$

The second takes the square root of each side; the square root of 25 which is either 5 or -5 because (+5) (+5) = 25 and (-5) (-5) = 25 Therefore x = 5 or -5. The answer is usually written as $x = \pm 5$.

Example 2:

Solve the following quadratic equation

$$(x + 3) (x - 5) = 20$$

Multiply out the equation to form the quadratic equation as follows.

$$x^2 - 2x - 15 = 20$$

$$2^2 - 2x - 35 = 0$$

$$(x-7 (x+5) = 0$$

either
$$x - 7 = 0$$
 or $x + 5 = 0$

$$x = 7 \text{ or } x = -5$$

Example 3:

Solve the following quadratic equation:

$$8x^2 + 6x = 9$$

$$8x^2 + 6x - 9 = 0$$

Find the factors (2x + 3)(4x - 3) = 0

:- either
$$2x + 3 = 0$$
 or $4x - 3 = 0$

$$2x = -3$$
 or $4x = 3$

$$x = \frac{3}{2}$$
 or $\frac{3}{4}$

SELF-ASSESSMENT EXERCISE

Solve the following equations

i.
$$x^2 - 6x + 9 = 0$$
, ii. $x^2 - 5x - 6 = 0$, iii. $x^2 + 9x + 14 = 0$

3.2 Solution by Completing the Square

This involves a process of converting the equation into perfect square and taking the root of each side. Example: to convert $x^2 + 6x$ into a perfect square we add to it $(\frac{1}{2} \text{ of } 6)^2 = 3^2$ because $x^2 + 6x + 3^2 = (x + 3)^2$ similarly to convert $y^2 - 7y$ into a perfect square, we add to it $(\frac{1}{2} \text{ of } 7)^2 = (\frac{7}{2})^2$, because $y^2 - 7y + (\frac{7}{2})^2 = (y - \frac{7}{2})^2$

Generally, equation $y^2 + bx$ becomes a perfect square if we add $(\frac{1}{2}b)^2$ to the equation $y^2 + by + (\frac{1}{2}b)^2 = (y + \frac{1}{2})^2$

Example 4:

Solve the following equation by completing the square y^2 - 6y = 27. Add 3^2 to each side of the equation $y^2 + 6y + 3^2 = 27 + 9$ $(v + 3)^2 = 36.$

Take the square root of each side: the square root of 36 is either + 6 or -

$$y + 3 = +6$$
 or $y + 3 = -6$
 $y + 3 = -6$

Example 5:

What should be added to $y^2 + 6y$ to make the expression a perfect

Suppose $y^2 + 6y + k$ is a perfect square, and that it is equal to $(y + a)^2$. It is known by expansion that $(y + a)^2 = y^2 + 2ay + a^2$ therefore $y^2 + 2ay$ $+a^2$ and $y^2 + 6y + k$ are identically equal. If we compare the coefficient of y,

$$2a = 6$$

$$- a = 3.$$

:-
$$a = 3$$
.
Therefore $y^2 + 6y + k = (y + 3)^2$
= $y^2 + 6y + 9$.

This shows that 9 should be added and k equals 9. Then the equation is $y^2 + 6y + 9 (y + 3)^2$. In practice the quantity to be added is the square of half of the coefficient of y (or any other letter that may be involved in example 5 above. The coefficient of y is 6, half of 6 is 3, and the square of 3 is 9 that is why 9 should be used to make it a perfect square.

Example 6:

Solve the equation by completing the square $y^2 - 8y + 3 = 0$.

The left hand side of the equation does not factorize, therefore the equation is first rearranged to make the left hand side a perfect square.

$$y^2 - 8y + 3 = 0$$

Subtract 3 from both sides

$$y^2 - 8y = -3$$

Add 16 to both sides of the equation

$$y^2 - 8y + 16 = -3 + 16$$

$$y^2 - 8y + 6 = 13$$

$$(y-4)^2=13$$

:-
$$y - 4 = \pm \sqrt{13}$$

$$y = 4 \pm \sqrt{13}$$

SELF- ASSESSMENT EXERCISE

- i. From the following add the term that will make each expression a perfect square.
 - $w^2 4w$
- (b) $v^2 7v$ (c) $x^2 + 5x$
- Solve the equation below ii.

 - (a) $x^2 + 18 = 9x$ (b) $x^2 + 10x + 21 = 0$ (c) $9y^2 + 6y + 1 = 0$

3.3 Solution to Quadratic Equation by Formula

Mathematically, any quadratic equation can be reduced to the form of expression as $ax^2 + bx + c = 0$. The formula for the values of x is often called almighty formula or the formula. It can be expressed as follows.

$$\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Example 7:

Solve the quadratic equation $5x^2 = 9x = 6$. The equation $ax^2 + bx + c = 0$ is equivalent to $5x^2 - 9x - 6 = 0$ through rearrangement. Therefore a=5, b=-9, c=-6. It can now be substituted into the formula as follows.

$$x = -(-9) \pm \sqrt{(-9)^2 - 4(5)(-6)}$$

$$2(5)$$

$$= 9 \pm \sqrt{(81 + 120)}$$

$$10$$

$$9 \pm \sqrt{201}$$

$$10$$

$$x = 9 \pm 14.8$$

$$10$$

$$x = -23.18$$
or
$$-5.18$$

$$10$$

$$= 2.318$$
or
$$-0.518$$

SELF-ASSESSMENT EXERCISE

Solve the following equation:

i.
$$x^2 + 7x = 5$$
 ii. $5x^2 - 7x - 4 = 0$ iii. $2x^2 - 5x = 4$

3.4 Solution by Graph

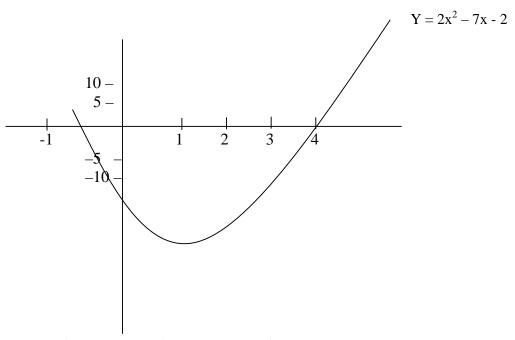
Quadratic equation can be solved using graphical method. In this method, the equation and the range for the graph would be given.

Example 8:

Given the following quadratic equation $y = 2x^2 - 7x - 2$, draw a graph for values of x range from -1 to +4.

The first step is to make a table, work by rows $y = 2x^2 - 7x - 2$

X	-1	0	1	2	3	4
-2	-2	-2	-2	-2	-2	-2
$2x^2$	2	0	2	8	18	32
-7x	7	0	-7	-14	-21	-28
$Y=2x^2-7x-2$	7	-2	-7	-8	-5	2



Scale 2cm on x-axis represent 1 unit. 5cm on y a-axis represent 1 unit

The solutions are at point 'A' and 'B'. It can be read to determine the actual points that are optimal.

SELF-ASSESSMENT EXERCISE

Draw the graph of y where $y = 4x^2 + 6x - 7$ for values of x range from -3 to +2

4.0 CONCLUSION

In this unit, you have learnt that quadratic equation can be solved using different methods so as to enrich our knowledge of algebra in business and planning. You have seen how important it is to use different options in solving the same problem. It could be in your business or daily transactions.

5.0 SUMMARY

The unit has thrown more light on the operations of quadratic equations using completing the square, factorization, formula and graphic methods. Any of the methods will give the same solution. However the choice is for you to determine the approach that you understand best.

6.0 TUTOR-MARKED ASSIGNMENT

- Given the following quadratic equation; $y = 2x^2 3x 7 = 0$ 1. using the range of x = -1 to +4, plot the graph and read the roots.
- Solve the quadratic equation $3y^2 4y + 5 = 0$ 2.
 - Solve the equation $y^2 4y + 13 = 0$ Solve the equation $x^2 7x + 10 = 0$ (b)
 - (c)

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UNIT 4 PROGRESSIONS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Arithmetic Progression Concepts
 - 3.2 Computation of the nth Term and Common Difference
 - 3.3 Computation of the Sum of Arithmetic Progression
 - 3.4 Geometric Progression
 - 3.5 Applications of Progression
- 4.0 Conclusion
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1.0 INTRODUCTION

A progression is a set of numbers in some definite order in successive terms or numbers of a sequence formed according to a given number of rules or conditions. The progression at any given time is an integer, a real number. The number could be positive or negative depending on the circumstance and the question that would be solved.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain arithmetic progression
- discuss geometric progression
- discuss the application of progressions.

3.0 MAIN CONTENT

3.1 Arithmetic Progression Concepts

An arithmetic progression is a sequence in which quantities increase or decrease by a common difference. The sequence 5, 7, 9, 11 ... n is an arithmetic progression since the difference between any two consecutive terms is 2. The sequence 3, 7, 11, 15, 19, 23 ... n is an arithmetic progression where the difference between any two consecutive terms is 4. Arithmetic progression occurs in the form of negative integers such as -2, -5, -8, -11 ... n, as the difference between any two consecutive term

is -3. An arithmetic progression can also have a combination of positive and negative integers such as 14, 8, 2, -4, -10, -16.

Given the sequence 1, 3, 5, 7 ...n, you would observe that there is a rule governing the sequence as each number other than the first can be obtained from the preceding one by adding a fixed number 2. Each number or quantity in a progression is called a term, the difference between one term and the preceding one is called common difference which is denoted by 'd'. The first term in a progression is conventionally denoted by 'a'.

The terms generally of an arithmetic sequence can be written as, a, a + b, a + 2d, a + 3d ... a + (n-1) d. Therefore the nth term of an arithmetic progression is given by t = a + (n-1)d

The sum of an arithmetic progression (ap) is given by

$$Sn = {}^{n}/_{2} \{2a + (n-1) d\}$$

Or

$$Sn = {}^{n}/_{2} (a + L)$$

Here, L is the last term in the arithmetic progression.

3.2 Finding the nth Term of Arithmetic Progression

Based on the introduction, we stated that the nth term of an arithmetic progression can be computed as: $S_n = a + (n-1)d$. where 'a' is the first term, 'd' = common difference.

Example 1:

Find which term is 383 from the following series, $5 + 8 + 11 + \dots n$.

Solution

Based on the series the first term 'a' = 5, the common difference 'd' = 3, the nth term = 383

$$t = a + (n - 1) d$$

Substitute the variables

$$t = 5 + (n - 1)3 = 383$$

$$5 + 3n - 3 = 383$$

$$5 - 3 + 3n = 383$$

$$2 + 3n = 383$$

$$3n = 383 - 2$$

$$3n - 381$$

Divide through by 3
$$\frac{3n}{3} = \frac{381}{3}$$
$$n = 127.$$

Example 2:

In an arithmetic progression, the third term is 10 the 7th term of this progression is 34. Find the first term and the common difference.

Solution The first term = a, the common difference = d

Therefore the
$$3^{rd}$$
 term equation is = $a + 2d = 10$ (1)

The
$$7^{th}$$
 term equation is $= a + 6d = 34 \dots$ (2)

Solve the equations simultaneously

$$a + 2d = 10$$

$$a + 6d = 34$$

$$4d = 24$$

$$4d = 24$$

$$\frac{4d}{4} = \frac{24}{4}$$

$$d = 6$$

Substitute d = 6 in equation (1) we have

$$a + 2(6) = 10$$

$$a + 12 = 10$$

$$a = 10 - 12$$

$$a = -2$$
.

SELF-ASSESSMENT EXERCISE

How many terms of the series 24, 20, 16 should be so that the sum may be 72?

3.3 Computation of the Sum of Arithmetic Progressions.

Example 3:

Find the sum of the first 28 terms of an arithmetic progression whose series is given as $3 + 10 + 17 + \dots$ n

$$Sn = {}^{n}/_{2} \{2a + (n-1) d\}$$

$$= 14 \{6 + (27)7\}$$

$$= 14 \{6 + 189\}$$

$$= 14 (195)$$

= 2730.

SELF-ASSESSMENT EXERCISE

Find the sum of the first 42 terms of an arithmetic progression whose first term is 3, and the common difference is 7.

3.4 Geometric Progression

If the consecutive terms of a sequence differ by a common ratio, the terms are said to form a geometric progression. In other words, this is a type of progression in which one term other than the first can be

obtained from the preceding one by multiplying or dividing by a constant quantity known as the common ratio denoted by 'r' The first term of a geometric progression is conventionally denoted by 'a'.

The general form of geometric progression is given by as, a, ar, ar^2 , ar^3 ar^{n-1}

The nth term of a geometric progression is given by the formula $GP_n = ar^{n-1}$

Example 4:

If the third term of a geometric progression is 20 and the seventh term is 320, what is the first term and the common ratio?

The Third Term
$$= Gp_3$$
 is $ar^{3-1} = 20$
= $ar^2 = 20$ (1)

The Seventh Term = is $ar^{7-1} = 320$

$$Ar^6 = 320.....$$
 (2)

Divide equation (2) by equation (1)

$$\frac{\operatorname{ar}^6}{\operatorname{ar}^2} = \frac{320}{20}$$

$$r^4 = 16$$

$$r = \frac{4}{16}$$

$$r = 2$$

Substitute for r in equation (1)

$$ar^2 = 20$$

$$a2^2 = 20$$

$$a4 = 20$$

$$a = 20/4 = 5$$

Find the sum of the geometric progression, given that the geometric progression series is: $Sn = a + ar + ar^2 + ar^3 + ... + ar^{n-1}$ (1)

Multiply through by r the common ratio

$$rsr = ar + ar^2 + ar^3 + ar^4 + ... + ar^n$$
 (2)

Subtract (1) from (2)

$$Sn = rsn = a - ar^n$$

$$S(1-r) = a(a-r^n)$$

$$Sn = \underbrace{a(1 - r^n)}_{1 - r}$$
 Used when $r < 1$

Sn
$$\underline{a(r^n - 1)}$$
 Used when $r > 1$

Example 5:

The second and third term of a geometric progression are 16 and 64 respectively, find the first term and the common ratio of the progression.

The second term ar = 16

The third term $ar^2 = 64$

$$\frac{ar}{16} = \frac{ar+2}{64} \\
 \frac{64}{16} = \frac{ar^2}{ar} \\
 4 = r \\
 r = 4$$
But the second term $ar = 16$
 $a(4) = 16$
 $a = 16/4$
 $a = 4$

Example 6:

The third term of a geometric progression is 20 and the seventh term is 320, what is the sum of its first nine terms? Gp_3 :- $ar^2 = 20$

$$Gp_4:- ar^6 = 320$$

$$ar^6 = 320$$

$$ar^4 = 16$$

$$r = 2$$
Substitute and find common ratio
$$ar^2 = 20$$

$$a2^2 = 20$$

$$4a = 20$$

$$A = 20/4 = 5$$

$$Sn = a(r^n - 1) = 5(2^9 - 1) = 5(512 - 1)$$

$$r - 1 = 2555$$

3.5 Applications of Progressions

A man starts work with an annual salary of \$14, 000 and received annual increase of \$480 a year (a) How much did he receive for the first four years. (b) How much will he receive in the tenth year of employment?

$$\begin{split} Ap_n &= a + (n-1)d \\ Ap_1 &= 1400 + (1-1) \ 480 \\ 1400 + (0) \ 480 \\ &= 14,000. \\ Ap_2 &= 14,000 + (2-1) \ 480 \\ &= 1400 + (1) \ 480 \\ &= 1400 + (1) \ 480 \\ 14480. \\ Ap_3 &= 1400 + (3-1) \ 480 \\ &= 1400 + (2) \ 480 \end{split}$$

```
1400 + 960
14960.
Ap_4 = 1400 + (4 - 1) 480
1400 + (3) 480 1400 + 1440
= 15440
The total amount for the first four years will be 1400 + 14480 + 14960 + 15440 = \underline{58880}
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b. In the tenth year n = 10

Ap_{10} = 1400 + (10 - 1) 480

1400 + 9 (480)

1400 + 4320

18320
```

SELF-ASSESSMENT EXERCISE

Umenemi was employed earning \$\frac{\text{N}}{12}\$, 000 annually. He is offered a choice between a yearly increment of \$\frac{\text{N}}{150}\$ and an increment of \$\frac{\text{N}}{420}\$ every two years. Calculate the total sum he will earn in the course of 20 years under each option offered to him.

4.0 CONCLUSION

In this unit, you have seen that arithmetic and geometric progression form an integral part of business mathematics. It equally has wide applications in business and economics. Therefore it is very essential for you to get involved in the learning of progressions as a means of enhancing quantitative reasoning.

5.0 SUMMARY

In this unit, the meaning and scope of progressions have been examined. The concept of arithmetic progression, geometric progressions and the process of deriving the formula and applications of the equations were used in the progressions. The application of progression in business life was also highlighted and illustrated to give a balanced knowledge of the unit.

6.0 TUTOR-MARKED ASSIGNMENT

1. Find the values of x, y, z if 12, x, y, z, -4 form an arithmetic progression.

- 2. (a) The third term of an arithmetic progression is 42 and the 13th term is 182, find the first term and the common difference.
 - (b) A man was employed with an annual salary of №280, 000 and receives an annual increment of №820 (i) How much does he receive for the first three years?

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MODULE 3 STATISTICAL INVESTIGATION AND DATA COLLECTION

Unit 1	Statistical Investigation and Data Collection
Unit 2	Data Presentation in Statistics
Unit 3	Measures of Central Tendency
Unit 4	Measures of Dispersion

UNIT 1 STATISTICAL INVESTIGATION AND DATA COLLECTION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Descriptive and Inferential Statistics.
 - 3.2 Statistical Enquiries.
 - 3.3 Data Collection Strategies.
 - 3.4 Sampling in Data Collection
 - 3.5 Problems and Solutions in Data Collection.
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
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1.0 INTRODUCTION

Statistics is a process of factual data collection and analysis of the collected data. It involves collection of numerical facts in a systematic way. Statistics also involves the careful analysis of the data collected in form of tables and the interpretation of such data. It also involves the use of scientific method of collecting, organising, summarising, presenting and analysing data as well as drawing conclusions so as to make you take reasonable decisions concerning a given phenomenon.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the basic concept of descriptive and inferential statistics
- discuss strategies of statistical enquiries
- discuss and apply the data collection method in statistics.

3.0 MAIN CONTENT

3.1 Descriptive and Inferential Statistics

(1) **Descriptive Statistics**

In descriptive statistics, the data collected describes the situation that existed at the point the census was taken. It provides a step by step detail of data available and collected at any given period. The important characteristic of descriptive statistics is that the population to be studied is included. If the University of Jos, for example, is taken as a point of discussion, the Vice Chancellor should know the number of deans of faculties, the heads of departments and the heads of units who are regarded as administrative instruments in the institution. The deans should know the number of heads of departments and other staff members. The knowledge of the categories of staff is also important. So also does it apply to the head of department and lecturers who need to know the number of students and the score of each student in each course. Events are described as they happen and these could be presented in:

- i. bar charts
- ii. pie charts or in pictorial form.

Supposing in the faculty of social sciences, each student is allocated a file and all information required of them is included in these files, a lot of things can be done with the information. If we want to consider the scores of female and male students statistically, we can draw graphs to represent the information. If we are concerned with the relationship of individual scores to the averages that have been computed, we can change the raw scores to standard score. All of the foregoing operations are included in what is referred to as descriptive statistics.

Descriptive statistics presents information in a convenient format usable and understandable in words with little figures included in the description of the data.

(2) Inferential Statistics

This is a data collected and used to make inference as related to the occurrence of events. Inferential statistics, which is mostly linked with probability theory, involves estimate outcomes of events. According to "Philips (1980), "it is that measure you have gotten that could have occurred by chance". Therefore statistics of inference essentially has to do with the measurement of chance.

We usually start with setting up a hypothesis or a number of hypotheses specifying our assumption(s) at the beginning of a study. For example, we can say that most members of the PDP are conservative in respect of economic policies while ANPP members take a liberal approach to economic policies. We are able to make these assumptions from selected samples of members of the two parties. Sometimes, this hypothesis may turn out to be true and it may also be false. When this happens, it is not possible to generalise because we may not be correct and a problem might arise.

This problem has been acknowledged in statistics and what to do is to use the most appropriate method measurement/analytical procedure so that the result we get will approximate the real population characteristic. Making inferences is a question of chance. However there are methods available for us to determine whether the results we obtain from a statistical investigation could be attributed to a chance occurrence even if its opposite is generally true. On the other hand, we could also measure the odds that the result of our investigation is false. This will place us in a position to make right conclusion on a particular social or political phenomenon. When we do this, there is to a certain measure the possibility of the truth or falsehood of our result which will be accepted to measure or test the statistical significance of our result.

Inferential statistics can be divided into two namely:

- deductive statistics
- inductive statistics
- (A) **Deductive Statistics. It** is the act of drawing inference about a sample using our knowledge of the population. The process involves arguing from the general (the population) to specific (the sample). It is deductive inference when probability of an event within a population context is obtained from a prior knowledge of the parameters of the distribution.
- (B) Inductive Statistics. It is the process of drawing inference about the population from the sample. It is arguing from the specific to the general. Reason of cost, time factor, accuracy and other constraints may make a complete enumeration (census) of the population impossible. The alternative is the use of concepts in probability to draw a sample from the population to obtain an estimate of the population parameters and test statements (hypothesis) about the parameters.

(C) Correlation Statistics: This is a statistical method that involves a comparison between two events. For example, the first semester test scores in statistics can be compared with the second semester test scores in year two. In statistics, this study of prediction is referred to as "Regression Analysis" the results of correlation analysis are used to study the reliability and validity of the test. Correlation analysis is a major part of statistical methodology.

SELF-ASSESSMENT EXERCISE

Explain inductive and deductive statistics.

3.2 Statistical Inquiries

The businessman or indeed anyone who has to administer any organisation is concerned with inquiries of many kinds. Some of these are capable of being treated and tested statistically and statistical evidence can be provided in respect of the information desired in any given situation.

The steps in a statistical inquiry are as follows.

1. The problem should be clearly stated

It is necessary to know the purpose of the investigation as this will influence the type of information to be obtained. Suppose the problem concerns wages in a factory, is it about wages earned or wage grade? Does the statistics concern all employees or only (women or men)? Should lost time, overtime, and bonus payments be included or allowed for? Should receipts in kind be included in the wage? The purpose of the investigation will provide guidance as to the exact information that ought to be obtained.

2. Selection of the Sample

In most problems concerning the administration of business, governmental or personal affairs or in making scientific generalisation, complete information cannot be obtained. Hence incomplete information must be used and this means taking the sample. The size of the sample and the sampling method will have to be determined. The best example of a sample inquiry in business is market research.

3. **Drafting the Questionnaire**

This is quite a difficult job if answers are to be of value. Usually, some questions have to be drafted to get the exact information needed for a

given time. A pilot survey is useful to enable a satisfactory questionnaire to be obtained. A great deal of information in business however is already available in form of accounting records, costing and administrative information about personnel. Questionnaires, apart from market research are therefore useful only for special inquiries.

4. **Data Collection**

Where not available as administrative published records, the most satisfactory way to obtain information is by means of enumerators or postal questionnaires. The types of questions to be asked depends on whether postal questionnaires are going to be used or enumerators are to be sent out. Questions for postal questionnaires are usually simple and easy to understand but those questions given to enumerators may be complex: Because enumerators would be there themselves and would be able to explain it clearly. Enumerators are usually trained on how to ask the questions before they are sent to the field. They also understand the objective of the research so that they work with a clear vision and focus.

5. Editing the Schedule

Questionnaires require checking, sometimes coding and calculations made before conclusion can be drawn to pick relevant information from the study.

6. **Organisation of Data**

The items need to be counted or the value summed up either in quota or in various categories before they can be calculated.

7. Analysis and Interpretation

Before the information acquired can be used, it is analysed and then interpreted. This requires a sound knowledge of statistical methods and also a sound knowledge of the subject for which statistical evidence has been obtained.

8. **Presentation**

This might take the form of tables, charts and graphs that will give a picture of the data under study. The presentation of statistical data helps to give an idea of the outcome of the study.

9. Writing of the Report

This gives the result of the investigation and where necessary we make recommendations. Tables and charts usually play an important part in biz reports. Such reports are sent to government department, business (biz) organisation and private bodies to be used for planning.

3.2.1 Uses of Statistics and Statistical Information

Although statistics is a powerful tool for analysing numerical data, its application is widely seen in all fields of human endeavours. For instance we apply statistics in the following fields.

- A. **Physical Science**. It can be used to determine whether or not experimental results should be incorporated into the general body of knowledge.
- B. **Biological and medical science**. Statistics guides the researchers in determining the experimental findings that are significant enough to demand further study, or to be tested more to meet human needs. Thus the physician uses statistics to access the effectiveness of a particular treatment and statistics also helps the pharmacologists to evaluate a proposed drug. In some fields such as genetics, statistics is thoroughly integrated into the field to study the multiplication of cells and other variables.
- C. **Social Sciences**. The role of statistics in the social sciences cannot be ignored especially in business administration. Also in accounting, political science, psychology, sociology and economics. The behavior of individuals and organisations can be monitored through "numerical data" to lend credence to models and theories that are applicable to man.
- D. **Engineering, Education and Business**. The professional fields of engineering, education and business all employ statistics in planning, establishing policies and setting standards. The headmasters or principal of a school may use statistics to write the curriculum, the school enrolment and the teachers required. The civil engineer can use statistics to determine the properties of various materials and perform some durability tests. The company manager may employ statistics to forecast sales, design products and produce goods more efficiently.
- E. **Meteorology**. Statistical information is also used in meteorology i.e. the science of weather prediction. In fact, the application of statistical techniques is so widespread and the influence of

statistics in our lives and habits is so great that the importance of statistics can hardly be over emphasised. There can be little doubt then on the effect of statistics and statistical techniques on each of us. The result of statistical studies are seen but perhaps not realised.

In scientific and behavioral research, statistical tools enable success of research results. In business and economic situations, its use is highly appreciated. Below is the summary of some of the uses of statistics in everyday life:

- 1. For summarising large mass of data into concise and meaningful form leading to a better understanding of condensed data.
- 2. Giving visual impact on data especially when presented in diagrams and charts.
- 3. Enabling comparison to be made among various types of data
- 4. Making conclusions from data generated in pure experimental, social and behavioral research.
- 5. Enabling a business establishment to make accurate, reasonable and reliable policies based on statistical data.
- 6. Predicting future events in daily life and business.
- 7. For the formation as well as testing of hypothesis.
- 8. For prediction of Gross National Product (comparing it with that of other countries) input-output analyses, public finance and consumer finance.
- 9. For budgeting and planning
- 10. Widely used in industrial and commercial dealings as well as government establishment.
- 11. Enables one to understand relevant articles in scientific journals and books.

3.2.2 Problems of Data Collection

Data collection can be difficult or inaccurate sometimes. The absence and non-availability of accurate statistical data may be due to all or some of the following reasons.

- 1. Lack of proper communication between users and producers of statistical data.
- 2. Difficulty in estimating variables which are of interest to planners.
- 3. Ignorance and illiteracy of the respondent.
- 4. High proportion of non response due to suspicion on the part of respondent.
- 5. Lack of proper framework from which samples can be selected.

6. The wrong ordering of priorities including misdirection of emphasis and bad utilisation of human and material resources.

3.2.3 Limitations of Statistics

- 1. Statistical data or result is only an approximation of the total and therefore not entirely accurate in some cases. This is because not all the population will be covered for any sample study.
- 2. Statistics, if not carefully used, can establish wrong conclusion and therefore it should only be handled by experts. Where experts are not available, some form of training should be conducted for those that may be required to carry out any statistical research.
- 3. Statistics deals only with aggregate of facts as no importance is attached to individual items.

SELF-ASSESSMENT EXERCISE

- i. List and explain the steps of statistical enquiry.
- ii. List the uses of statistics and statistical information

3.3 Methods of Data Collection

Business data are collected in the normal cause of administration and not specifically for statistical purpose, however, there is no reason why records should not serve the two purposes and in such cases, care should be taken to ensure that the record is accurate statistically as well as administratively.

The following list covers some of the important methods of collecting data.

1. **Postal Questionnaire**

This takes the list of questions sent by post. Unless of course the respondent has an interest in answering it or is under legal compulsion, the postal questionnaire is generally unsatisfactory, producing few replies and those of a biased nature. The postal questionnaire is satisfactory when sent by trade associations to the members, since the members have interest in answering it. Some firms have tried to get answers by offering small gifts. This is not a very good idea since it will produce biased answers as the respondent tries to please the donor.

2. Questionnaire to be filled by the enumerator

This is the most satisfactory method. The enumerator or field workers can be briefed so that they understand exactly what the questions mean.

They get the right "answers" and they fill in the questionnaire more accurately than would be the respondent themselves.

3. **Telephone**

This involves asking questions by phone calls to the respondent. Asking questions by telephone is not usually a very good method. People who possess telephones form a biased sample. Telephone interviews are useful for certain kinds of radio research.

4. **Direct Observation**

This method entails sending observers to record what actually happens while it is happening at the current period. An example of this method being suitable is in the case of traffic census. Actual measurement or counting also comes under the heading of direct observation Examples occur in statistical quality control. It can either be participatory or non-participatory.

5. Report

This is a method that may be based on observations or informal conversations. These are usually incomplete and biased but in certain cases it may be useful.

6. **Results of Experiments**

This method is of interest to the production mangers, engineers, the agronomists and applied scientists. It requires carrying out experiments, sample tests, and laboratory examinations to determine the behaviour of certain events.

7. Interview

The researcher asks the respondent the questions listed. The listed questions provide a guide to what is being investigated. He or she also fills in the answers. The researcher now has the duty of obtaining accurate information from the respondent. The interviewer must record factually and accurately, all the answers given. The interviewer must be sure of interviewing the right person or sample.

8. **Personal Investigation**

Personal investigation involves the researcher using direct contact with the respondent. These tends to be time consuming, expensive and limited in size, but the data collected will be complete and reliable. It is useful for pilot survey.

9. **Team of Investigation**

It is the same as the personal investigation method. The only difference is that the investigators go to the respondents in a group. The group can cover a large field than the personal investigation, but it will be more expensive. The members of the team should obviously be carefully briefed to ensure that the data they collect is satisfactory. This method is sometimes called "delegated personal investigation".

10. **Registration Method**

This method involves the recording of vital events as they are taking place within a given time. Vital events include statistics on births, deaths, migration immigration, separation and adoption.

11. Panel Method

This method is commonly used in interviewing job seekers in Nigeria. Under this method, certain groups of people that are specifically trained interview certain people. This is to determine the true position of events under study.

SELF-ASSESSMENT EXERCISE

List and explain the various methods of statistical data collection.

3.4 Sampling Process in Data Collection

Instead of obtaining data from the whole of the material being investigated, sampling methods are often used, in which only the sample selected from the whole is dealt with and from these samples conclusions are drawn relating to the whole population. If conclusions are to be valid, the sample should be the representative of the whole, the selection of these samples should therefore be made with great care.

3.4.1 Reasons for Using Samples

- It is used in collecting data based on certain characteristics of a group of individuals or objects. It is often impossible or impracticable to observe the entire group, especially if it is large. Instead of examining the entire group called the population or universe, one examines a small part of the group called "a sample"
- 2. Even where complete inspection is possible, sampling may have economic advantages. Resources such as materials, time, personnel and equipment, are limited in any investigation. It is

then necessary to use the available resources to get necessary information by selecting a sample instead of the entire population.

3. Another reason for using sample is that, for making data, the population is inaccessible.

In any case, the sample chosen must be a representative of the whole population as the sample would provide information about the population characteristics which are being examined. The population refers to the whole of the material from which the sample is taken. The frame will consist of a list of all the items in the population or some means of identifying any particular item in the population. This frame is necessary so that any item in the population can be part of the sample. The frame must be complete, i.e. no item of the population should be left out and it should not be defective, because of being out of date or contain inaccurate, duplicate items or inadequate because it does not cover all the categories require to be included in the investigation.

3.4.2 Sampling Techniques

Some of the basic techniques used in statistical sampling include the followings.

1. Random Sampling The word "random" does not mean haphazard it refers to a definite method of selection. A random sample therefore is one in which every member of the population has an equal chance of being selected in the sample.

A technique for obtaining a random sample is to assign numbers or names to each member of the population. Write these numbers on small pieces of paper, place them in a box, and after mixing thoroughly, draw from the box in lottery fashion. Another method is to use a table of random sampling numbers when the random sample of names have been drawn, interviews or enumerators would be sent to the people to collect all the necessary information. Although random sampling is a long, expensive operation, it does give a reliable, unbiased picture of the whole population. This method is suitable where the population is relatively small and where the sampling frame is complete.

2. **Systematic Sampling:** For practical work, it is easier to select every item in a list of the population. This method is termed "systematic sampling". The 1st of the sample unit will be selected by some random process For instance, if the list comprises a population of say 25,000 and the sample required is 500, then the selection of every 50th item will yield the required sample.

Systematic sampling is not random because once the initial starting point has been determined, it follows that the remainder of the item selected for the sample are predetermined by the constant interval (i.e. 50). In random/systematic sampling, the samples are believed to be homogeneous.

- 3. **Stratified Sampling:** So far we assume that the population to be sampled consists of a single homogeneous group, i.e. people with the same characteristics. Where the population is heterogeneous, i.e. comprises men and women in different age a group, in different social circumstances or of different backgrounds, a stratified sample is taken. This is because people in different social groups will think differently from other groups. In stratified sampling, the population is divided into strata, groups or blocks of units in such a way that each group is as homogeneous as possible (hence, same characteristics). Each group, block or stratum is then sampled at random. The stratified sample would be representative of the whole population.
- 4. **Multi-stage Sampling:** This is where a series of samples are taken at successive stages. For instance, in the case of a national sample, the 1st stage will be to break down the sample into the main geographical areas. In the 2nd stage, a limited number of towns and rural districts in each of the states will be selected. In the 3rd stage, within the selected towns and rural districts, a sample of respondents allocated to each state is drawn. This may also involve the list in which certain households are selected and many more stages may be added.
- 5. **Quota Sampling:** To economist and business managers, time and cost are taken into consideration in sample data. For this reason, a method of sampling known as quota sampling is extensively employed by many organisations. The essence of quota sampling is that the final choice of the respondents to be interviewed lies with the interviewer, this of course introduces bias. The quotas are chosen so that the sample is representative of the population in a number of respects according to the controls chosen. The interviewer is instructed to carry out a number of interviews with individuals who conform to certain requirements. Some of the requirements often used are age, sex and social class.
- 6. **Cluster Sampling:** In this technique, the country is divided into small areas almost similar like multi-level sampling method. The interviewers are sent to the areas with instruction to interview every person they can find that fits the definition given.

Generally, cluster sampling is used when it is the only way a sample can be found.

3.4.2 Errors in sample Data Collection

It is believed that the larger the sample the smaller the error. There could be sampling bias because the sample is too small. In order to reduce bias, it is being approved that the sample should be large enough to provide clear information on the topic to be studied.

- 1. Errors due to bias: Deliberate selection can introduce bias in a sample. Substitution also introduces bias, failure to cover the sample (not being consistent) introduce errors. Haphazard selection is also prone to errors.
- **2. The Questionnaires:** In drafting questionnaires, direct answers like "Yes" or "No" should be minimal and the respondent given an opportunity to express his or her understanding.
- **3. Memory Error:** The respondent may give wrong information when the event being investigated has taken a long time. To minimize such errors, interviews should prevent asking questions on events that happened long ago, bringing events that will help the respondent recall would be of help in providing accurate information.
- **4. Coding Error:** There are circumstances where we may use a wrong code in the process of carrying out a statistical survey. This gives rise to errors in statistical data collection.
- **Editing error**: Sometimes errors emerge during coding; it may entail writing the wrong thing when compiling the result e.g. writing "1997" instead of "1977"
- **6. Error due to tabulation:** Sometimes, errors emerge as a result of wrong tabulation of statistical information.
- **Respondents' error:** Due to poor educational background and illiteracy, the respondent may give wrong information. Also because of ignorance or lack of understanding of the context of the questionnaire and sample questions, the respondent may give wrong information.

Error in sharing of questionnaires: There are situations where the individual carrying out the statistical survey may administer the questionnaire wrongly. For example a questionnaire meant for the

working- class may be given to the students. The students may not provide the work experience aspect to the questionnaire.

SELF-ASSESSMENT EXERCISE

List and explain the various methods of sampling in data collection.

4.0 CONCLUSION

In this unit you have learnt that statistics is a vital course that you should take seriously due to the wide application of the subject in all areas of life. It is therefore important for you to get involved in learning the subject. This will help you in the other courses.

5.0 SUMMARY

The unit has thrown more light on the meaning and scope of statistics. The concept of gathering statistical information and the merit of such a process were also analysed. Sampling, as a major strategy for data collection and the likely errors associated with it, were discussed. In the next study unit, you will be taken through the discussion on statistical data presentation.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. List and discuss the various techniques of statistical enquiries.
- 2. Mention and explain types of techniques used in data collection.

7.0 REFERENCES/FURTHER READING

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UNIT 2 DATA PRESENTATION IN STATISTICS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Data Presentation in Tables
 - 3.2 Data Presentation using Pie Charts
 - 3.3 Data Presentation using Bar Charts
 - 3.4 Data Presentation in Frequency Table and Graphs.
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor- Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

In statistical analysis, information gathered can be presented in the form of tables, pie chart, bar charts and frequency distribution. The presentation in the tabular form gives an idea of the distribution of the information gathered for further evaluation. The pie chart and the bar chart are pictograms that give a quick understanding of the statistical information obtained.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the basic statistical concepts with table
- present data in pie charts
- present data in bar charts
- present data in frequency table and graphs.

3.0 MAIN CONTENT

3.1 The Presentation of Statistical Data using Tables

In statistical analysis, it is often simpler and quicker to illustrate ideas with tables, charts and graph than with endless written pages. Statistical tables, graphs and diagrams are visual aids to a quick understanding of information. Such visual aids are condensed ideas which are more meaningful and comprehensive to readers who have difficulty in interpreting statistics from printed words or who have less time to read volumes. Visual comparisons become important in economic and business analysis and it is also an acceptable norm in modern writing.

It is an orderly arrangement of information showing the relationship between variables. Consider the following marks obtained by students in an entrance examination.

Example1:

- A's scores: English 40%, Mathematics 60%, General Knowledge 80%
- B's scores: English 80%, Mathematics 60%, General Knowledge 60%
- C's scores: English 80%, Mathematics 40% General Knowledge 60%
- D's scores: English 60%, Mathematics 50% General Knowledge 80%

The pieces of information do not make for easy comprehension.

However, we can prepare a table to show each student's marks under each subject as shown in the table below.

D C	CC				•
Performances	ot tour	students u	า ๑ท	entrance	examination
1 CHOI mances	OI IOUI	Students II	ı an	Citti ancc	CAammanon

Students	Subjects						
Students	Maths %	English %	Gen. Know %	Total marks			
A	60	40	80	180			
В	60	80	60	200			
С	40	80	60	180			
D	50	60	80	190			
Total Score	210	260	280	750			

From the table, we can see at a glance the relative performance of the four students. We can also interpret the relative performance in the 3 subjects. Based on the table presented, we can draw some inferences. The aggregate marks in the last column show that "student B" has the highest total marks (200%) while those for the subject show that General Knowledge has the highest total score (280 marks). We can state that the students performed best in the General Knowledge among the three courses, and the students' performance is lowest in Mathematics. We could not have seen this easily without the table.

Importance of Tables

- It is used to interpret data as shown in the table.
- Data in the table can be used for comparative analysis. Quick decisions
- Can be taken, based on information derived from the table.
- Information from tables occupies less space.

- It reveals at a glance, the information conveyed on the data.
- The data given can be used in forecasting the future performance of events.

SELF-ASSESSMENT EXERCISE

Consider the following information on the performance of some students in the post-UME examination. Matta, English 57%, Mathematics 50%, Current Affairs 81%. Idoko, English 87%, Mathematics 79%, Current Affairs 67%. Dawang, English 69%, Mathematics 62%, Current Affairs 61%. Present the performance of the students in a table. Interpret your findings.

3.2 Pie Charts

It is a circular representation of data and it is based on the fact that the sum of angles about a point is 360° . That means a pie chart is a pictograph drawn in a circle to represent relative performance of a given variable in relation to the total value. We know a circle has a total angle of 360° , the pie chart is constructed by dividing 360° proportionately. The information collected for analysis is converted in the 360° proportionally.

Example 2:

1. A pure water company awarded contracts to various contractors for constructing a factory house as follows.

25,000
20,000
5,000
10,000
60,000

Represent the above information on a pie chart.

Solution

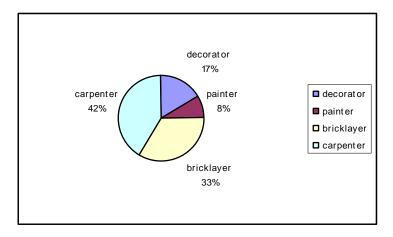
The first step is to find the total spent on the contract award, then divide each by the total and multiplied by 360° as follows.

Carpenter =
$$\frac{25000}{60000}$$
x 360 = 150⁰
Bricklayer = $\frac{20000}{60000}$ x 360 = 120⁰

Painter =
$$\frac{5000}{60000}$$
 x $360 = 30^0$

Decorator =
$$\underline{10,000}$$
 x $360 = 60^0$
 $60,000$

Pie Chart showing the proportion of contract awarded to the various Contractors



The pie chart shows that the carpenter received the highest allocation from the contract, this is followed by the bricklayer then the decorator and the painter had the lowest allocation.

Example 3:

The following information shows the contribution of a family in the upkeep of their school child in the first and second term at school.

2. Contributions to a student's pocket money

Contributors	1 st term(₩)	$2^{\text{nd}} \text{ term}(\frac{\mathbf{N}}{\mathbf{N}})$
Father	15	25
Mother	20	20
Friends	10	5
Total	45	50

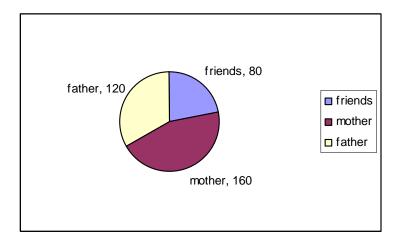
Present the information on a pie chart.

1st Term

Father =
$$\frac{15}{45}$$
 x 360 = 120⁰

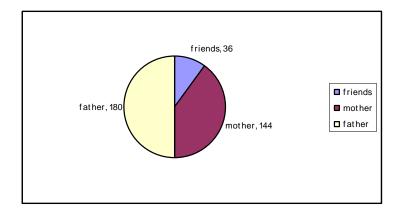
Mother =
$$\frac{20}{45}$$
 x 360 = 160⁰

Friends =
$$\frac{10}{45}$$
 x 360 = 80⁰



The pie chart shows graphically the contribution by the three individuals in the upkeep of the student. We can see from the diagram that the mother contributes more toward the upkeep than the father while the friends made the least contribution in the first semester.

$$2^{\text{nd}}$$
 term
Father = $\frac{25}{50}$ x $360 = 180^{0}$
Mother = $\frac{20}{50}$ x $360 = 144^{0}$
Friends = $\frac{5}{50}$ x $360 = 36^{0}$



Using the pie chart, in the second semester, the father contributed more toward the upkeep of the student than the mother and friends.

SELF-ASSESSMENT EXERCISE

Given the following information on the performance of a student in five courses registered in the university in the first year; Business Administration 86, Economics 72, Political Science 52, General Studies 49, Accounting 57. Use a pie chart to show the performance of the student in the first year.

3.3 The Bar Chart

It is a chart in which data is presented in the form of a bar and it is used to show magnitude, usually there are three types of bar charts as follows.

- 1. Simple Bar Chart
- 2. Component Bar Chart
- 3. Compound or multiple Bar Chart

A bar chart can also be defined as a series of rectangles which heights are plotted proportionally to the values that are being represented or assessed. The height of the bars should be plotted to scale to show relative measurement. The width of the rectangle could be of any size but all the bars must have the same width.

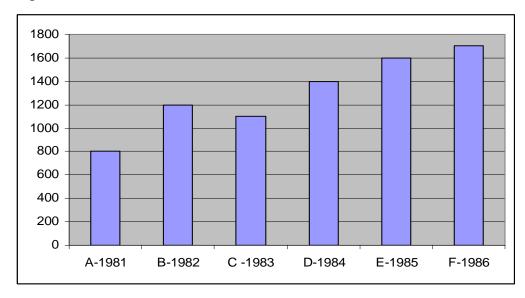
1. Simple Bar Chart

The simple bar chart is the chart of one or more bars in which the length of the bars indicate the magnitude of the data. Each shows the magnitude of the occurrence of the situation under study.

Example 4: Peace House has the following 6 years projection for those that will attend its annual teachers' conference. Present the data in a bar chart.

Year	Attendance	Year	Attendance
1981	800	1984	1400
1982	1200	1985	1600
1983	1100	1986	1700

Representation on the bar chart



Using the bar chart, it can easily be inferred that the highest attendance for the conference is 1986 and the lowest is 1981.

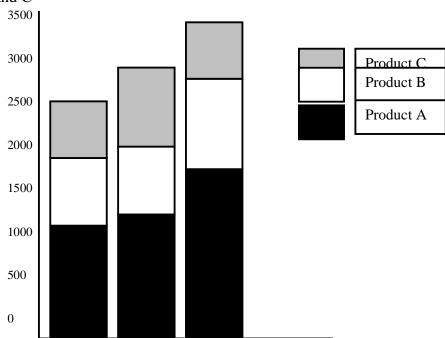
2. Component Bar Charts

A component bar chart shows the breakdown of the total values for given information into their component parts. There are three types of component bar charts.

- a. Multiple bar charts
- b. Bar charts studying relative value
- c. Percentage component bar chart

Example 5:Sam and Sam Ltd. have the following sales of 3 products in the market. Present the information on a component bar chart

	1997	1998	1999
	(sales)	(sales)	(sales)
Product A	₩1000	N 1200	₩1700
Product B	N 900	N 1000	₩1000
Product C	N 500	N 600	N 700
Total	N 2400	N 2800	N 3400



Sam and Sam Ltd. company component bar charts for products A, B, and C

3. Multiple Bar Chart

They are charts in which the component parts are represented separately to show the total values.

Example 6:

Suppose your father, mother and friends gave you N15, N20 and N10 respectively for your pocket money for the first term in school, the multiple bar chart of this sum is shown below.

Contributions to 1st Term Pocket Money

Here, the total sum contributed by the father, mother and friends is the sum of the heights of the rectangles, each portion is represented separately.

4. Percentage Multiple Bar Chart

This mainly shows relatively the values that are expressed as percentages of the totals. Although the total contributions for the two terms are different, to construct a percentage component bar chart, the information would be represented as 100%. Their components therefore will add up to 100%.

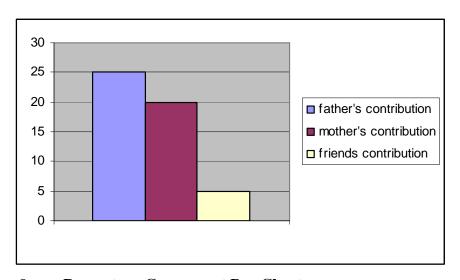
Example 7:

Given the following information on the contribution for a student upkeep, Father = $\frac{N}{25}$, Mother = $\frac{N}{20}$, Friends = $\frac{N}{20}$, present the data on percentage multiple bar charts. The first step is to add the total contribution, then divide each contribution by the total and multiply by 100% as follows.

$$\underline{25}$$
 x $100 = 50\%$

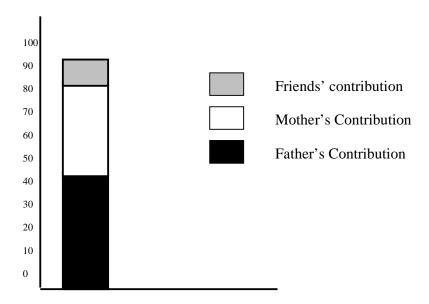
$$\frac{20}{50} \times 100 = 40\%$$

$$\frac{5}{50} \times 100 = 10\%$$



5 Percentage Component Bar Chart

This mainly shows the values expressed as percentages of the totals. In an attempt to construct a percentage component bar chart, the information would be presented as 100% while their component will add up to 100%.



SELF-ASSESSMENT EXERCISE

The following data shows the performance of a student in four courses in first semester. Economics 75, Sociology 63, Mathematics 48, Accounting 61.

- i. Present the information in a bar chart
- ii. Present the data in multiple bar charts
- iii. Present the information in a percentage component bar chart.

3.4 Frequency Distribution.

A frequency distribution is an array of numbers. The unorganised data collected during investigation is known as "raw data". You can also arrange the data in ascending or descending order. The data that is arranged in such order is called an array of data.

Example 8:

Given the following data on the number of vehicles that are parked daily in a parking lot; 92, 78, 68, 58, 45, 89, 75, 68, 58,45, 43, 57, 67, 75, 87, 42, 57, 65, 74, 87, 85, 73, 65, 56, 41, 36, 56, 61, 69,84,32, 55, 61, 69, 84, 81, 69, 60, 52, 32, 25, 49, 58, 69, 79, 15, 49, 58, 69, 79, arrange the data in an ascending order.

Solution is as follows:

15	25	32	32	36	41	42	43	45	45
49	49	52	55	56	56	57	57	58	58
58	58	60	61	61	65	65	67	68	68
69	69	69	69	69	73	74	75	75	78
79	79	81	84	84	85	87	87	89	92

From the array, we can easily identify the highest number as 92 and the lowest as 15. The difference between these numbers is known as the range i.e. 92 - 15 = 77 (The range is the highest value minus the lowest value). We can still organise the data further because it is not in categories or groups or classes, normally we expect that the classes should be between 5 & 20 or 11 & 20. In this case we are using 11 - 20, 21 - 30, 31 - 40 etc.

We use tallies to form the frequency table. Tallies are strokes used for counting and value for 5 tallies is denoted by four vertical strokes and one diagonal stroke (||||) this is to facilitate the counting. So for the strokes score, we obtain the following frequency table.

Classes	Tally	Frequency	Cum. F
11 - 20		1	1
21 - 30		1	2
31 - 40		3	5
41 - 50		7	12
51 – 60		11	23
61 - 70		11	34
71 - 80		8	42
81 – 90		7	49
91 – 100		1	50
		50	

Relative freq.	Cumulative	Class	Class
	relative freq.	Boundary	Mark
1/50 = 0.02	1/50 = 0.02	10.5-20.5	15.5
1/50 = 0.02	2/50 = 0.04	20.5-30.5	25.5
3/50 = 0.06	5/50= 0.1	30.5-40.5	35.5
7/50 = 0.14	12/50 = 0.24	40.5-50.5	45.5
11/50 = 0.22	23/50 = 0.46	50.5-60.5	55.5
11/50 = 0.22	34/50 = 0.68	60.5-70.5	65.5
8/50 = 0.16	42/50 = 0.84	70.5-80.5	75.5
7/50 = 0.14	49/50 = 0.98	80.5-90.5	85.5
1/50 = 0.02	50/50 = 1	90.5-100.5	95.5

Note: Class boundaries are used for histogram

- Class midpoint is obtained by $\underline{11+20}$ or using class boundaries

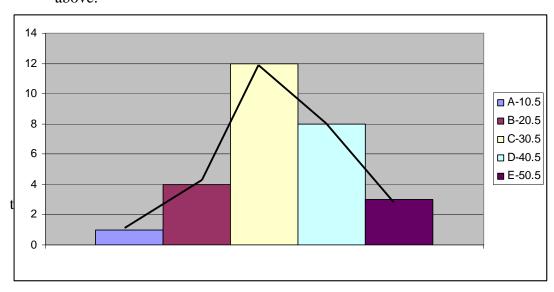
$$\frac{10.5 + 20.5}{2} = \frac{31}{2} = 15.5$$

Draw also the frequency polygon on the histogram

Group	1-10	11-20	21-30	31-40	41-50
Frequency	1	4	12	8	3
Class	0.5-10.5	10.5-20.5	20.5-30.5	30.5-	40.5-50.5
boundaries				40.5	

Solution

- We first calculate the class boundaries and obtain result as shown above:



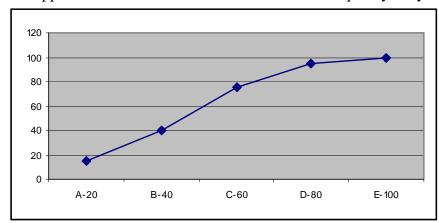
The rectangles can be used to form a line graph. The graph that results as shown above is called a frequency polygon.

A frequency polygon is a line graph of class frequencies obtained by connecting the midpoints of the tops of the rectangles forming the histogram (the graph of the cumulative frequency distribution is called an "Ogive").

Example 9:

Frequency distribution of marks of a class test

Marks	F	Cum. F
1-20	15	15
21-40	25	40
41-60	35	75
61-80	20	95
81-100	5	100
	100	



Use upper class marks for x axis and cumulative frequency for y axis.

SELF-ASSESSMENT EXERCISE

Given the following data on the number of vehicles that cross a traffic point daily; 92, 78, 68, 58, 45, 89, 75, 68, 58,45, 43, 57, 67, 75, 87, 42, 57, 65, 74, 87, 85, 73, 65, 56, 41, 36, 56, 61, 69,84,32, 55, 61, 69, 84, 81, 69, 60. present the information in a frequency table.

4.0 CONCLUSION

In this unit, you have learnt that pie and bar charts form an important part of statistical analysis. They equally have wide applications in business and economics. Therefore, it is very essential for you to get involved in learning data presentation in tables, pie and bar charts as a means of enhancing quantitative reasoning.

5.0 SUMMARY

The unit examined the meaning and scope of tables, pie and bar charts. Illustrations were also provided on how to compute tables, pie chart, bar chart and frequency polygon. Their applications in business life were also highlighted and illustrated to give an adequate knowledge of the unit.

6.0 TUTOR-MARKED ASSESSMENT

1. The followings show the number of stores that purchase Swan Water in Jos, Abuja, Akure, and Calabar respectively.

Jos 25,000 Abuja 20,000 Akure 10,000 Calabar 5,000

Present the above purchases of Swan water information on a pie chart

2. Agada foods is projecting the demand for its product as follows. Present the data in a bar chart.

Year	Demand	Year	Demand
1981	800	1984	1400
1982	1200	1985	1600
1983	1100	1986	1700

7.0 REFERENCES/ FURTHER READING

Frank, O & Jones, R. (1993). Statistics. London: Pitman Publishing.

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UNIT 3 MEASURES OF CENTRAL TENDENCY

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 The Mean
 - 3.2 The Median
 - 3.3 The Mode
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

The mean, median and mode are measures of central tendency, showing the average, the central number and the most frequent occurrence variable. Therefore the relationship between these three statistical variables is very important in the understanding of the unit. The descriptive statistics is aimed at describing data through summarising the values in the data set. One method of doing this is by finding a single value that will describe the general notation of the data. This single value, which is the central point of the distribution, is known as a measure of central tendency or location. Measures of central tendency are typical and representative of a data set. Every value in the distribution clusters around the measures of location. The population average, which in statistics is called the arithmetic mean, is of such measure. Others are the median and the mode.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the basic statistical concept of the mean and its illustrations
- discuss the median
- discuss the mode.

3.0 MAIN CONTENT

3.1 The Mean

The measure of central tendency most widely used is the 'Arithmetic mean' usually shortened as the 'mean'. For raw data, i.e. ungrouped data, the mean is the sum of all the values divided by the total number of values. To find the mean for example, we use the following formula.

Sample mean $(\overline{X}) = \underline{\text{sum of all values in the sample}}$ No. of values in the sample

Symbolically, it is $\overline{X} = {}^{\Sigma x}/_{n}$

Where \overline{X} = sample mean read as 'x bar'

 \overline{X} = a particular value

 Σ = sigma indicating addition

 Σx 's = the sum of all the x's

N = total number of values in the sample

The mean of a sample or any other measure based on sample data is called 'A statistic' i.e. A measurable characteristics of a sample.

Example 1:

1. The net weights of the contents of 5 coke bottles selected are: 85.4, 84.9, 85.3, 85.4, and 85.0 from the production. What is the arithmetic mean weight of the sample observation?

Solution

$$\frac{\overline{X}}{\overline{X}} = \frac{\sum X}{n}$$

$$= \frac{85.4 + 84.9 + 85.3 + 85.4 + 85.0}{5}$$

$$\overline{X} = 426/5 = 85.2$$

$$= 85.2 \text{kg}$$

The mean weight is 85.2kg

Many studies involve all the population values. The mean of the population in terms of symbols is $\mu = \frac{\Sigma x}{n}$ where

 $\mu = \frac{\sum x}{n} = \text{population mean}$

N = total number of observations in the population

As noted earlier, a measurable characteristic of a sample is called the statistic. Any measurable characteristic of a population such as the mean is called a parameter. A sample statistics is used to estimate a population parameter.

Properties of the Arithmetic Mean

As noted earlier, the arithmetic mean is a widely used measure of central tendency. It has several properties which include the followings.

- 1. Every set of interval level and ratio level data has a mean.
- 2. All the values are included in computing the mean.
- 3. A set of data has only one mean: It is unique
- 4. The mean is a very useful measure for comparing 2 or more populations.
- 5. The arithmetic mean is the only measure of central tendency where the sums of the deviation of each value from the mean will always be zero (0). Expressed, symbolically $\Sigma (x \overline{X}) = 0$

Example 2:

Find the Mean of 3, 8, 4

$$\overline{X}$$
 = $\frac{3+8+4}{3}$ = $\frac{15}{3}$ = 5
Deviation = $(3-5)+8-5)+(4-5)$
= $-2+3+(-1)=0$
= $-2+3-1=0$

Weighted Mean

If Masco Company pays its sales people either N6.50k, N7.50K or N8.50k an hour, it might be concluded that the arithmetic mean hourly wage is N7.50k. Found by the sum of \overline{X} , it gives us the following:

$$\frac{\text{N6.50k} + \text{N7.50k} + \text{N8.50k}}{3} = 7.50k$$

However, this is true only if there is the same number of sales people, earning N6.50k, N7.50k and N8.50k an hour. Suppose 24 sales persons earn N6.50k an hour, 10 are paid N7.50k and 2 get N8.50k. To find the mean, N6.50k is weighted or multiplied by 14 (6.50 x 15); N7.50 is weighted by 10(7.50x10); N8.50 is weighted by 2 (8.50 x 2). The resulting average is called the weighted mean.

In general, the weighted mean of a set of numbers designated x_1 , x_2 , x_3 ... x_n with corresponding weighted, w_1 , w_2 , w_3 ... w_n is computed by

$$\overline{X}_{w} = \underline{w_{1}}\underline{x_{1}} + \underline{w_{2}}\underline{x_{2}} + \underline{w_{3}}\underline{x_{3}}.... + \underline{w_{n}}\underline{x_{n}}$$

$$\overline{W_{1}} + \overline{W_{2}} + \overline{W_{3}}.... + \overline{W_{n}}$$

This may be shortened to

$$\overline{X} w = \underline{\Sigma(w.x)} \\ \underline{\Sigma} w$$

Example 3:

Masco Company pays its sales people either N6.50k, N7.50K or N8.50k. The corresponding weight is 14, 10 and 2 respectively. Determine the weighted mean?

$$\overline{X}$$
 w = $\frac{14 \times N6.50k + 10 \times N7.50k + 2 \times N8.50k}{14 + 10 + 2}$
= $\frac{41 + 75 + 17}{26} = \frac{183}{26}$
 \overline{X} w = N7.038
= N7.04K

SELF-ASSESSMENT EXERCISE

Bata Nig Ltd pays the following to its workers: 2000, 3789, 4302, 2118, 5002, determine the mean wage for the workers.

3.2 The Median

When the data contains one or two very large or very small values, the arithmetic mean may not be representative. The centre point for such problems can be better described using a measure of central tendency called "A median". For ungrouped data, the median is the midpoint of the values after they have been ordered from the smallest to the largest. There are as many values above the median as there are below it in the data array.

Example 4:

Given the following set of numbers 60, 65, 70, 80, and 275, the median is 70. In this example, there is an odd number observation "5". For an even number, the observations are ordered. The usual practice is to find the arithmetic mean of the two middle observations. Note that for an even number observation, the median may not be one of the given values.

Example 5:

A sample of a paramedical fees charged by U.J's clinic revealed these amounts; N35, N29, N30, N25, N32, N35. What is the mean charge?

Solution Arrange the paramedical fees from lowest to highest charge. 25, 29, 30, 32, 35, 35

The median fee is N31, found by determining the arithmetic mean of the observation at the centre i.e.

$$\frac{30+32}{2} = \frac{62}{2} = 31$$

An easy way to locate the position of the middle items for raw ungrouped data is by the formula;

Location of the median value = $\frac{n+1}{2}$

Where n = total no. of items

For the example above, there are 6 items, so $^{n+1}/_2 = ^{6+1}/_2 = ^{7}/_2 = 3.5$. After arranging the data from the lowest to the highest, we locate the middle item by counting to the 3.5^{th} item and then determine its values.

Properties of the Median

- 1. It is unique i.e. like the mean, there is only one median for a set of data.
- 2. To determine the median, arrange the data from lowest to highest and find the value of the middle observation.
- 3. The median is not affected by extremely large or small values, therefore, it is a valuable measure of central tendency.
- 4. It can be computed for ratio, interval level and ordinal level data.

SELF-ASSESSMENT EXERCISE

Samples of fees paid by commuters from Ikeja to Ogba are as follows: $\mathbb{N}150$, $\mathbb{N}120$, $\mathbb{N}130$, $\mathbb{N}120$ and $\mathbb{N}100$. Find the median values for the fees.

3.3 Mode

It is another measure of central tendency. It is the value of the observation that appears most frequency.

Example 5

Given the following set of data;

35, 49, 50, 50, 40, 58, 50, 60, 50, 65, 50, 71, 50, 55, what is the mode?

Solution

A look at the values reveals that 50 appears more than any other. The mode is therefore 50.

We can determine the mode for all levels of data whether they are nominal, ordinal, interval or ratio. 1). The mode also has the advantage of not being affected by extremely high or low values like the median. 2). It can be used as a measure of central tendency for open-ended distributions. The mode has a number of disadvantages however, that causes it to be used less frequently than the mean or median. For many sets of data, there is no mode if no value appears more than once.

SELF-ASSESSMENT EXERCISE

The following information represents the distribution of pencils in a market 40, 51, 39, 53, 51, 85, 75, 53, 44, 53, 90, 53. Find the mode of pencil distribution.

3.4 The Mean, Median and Mode of Grouped Data

In a grouped data, the information is presented in the form of a frequency distribution. It is usually impossible to secure the mean at face value using the original raw data. Thus if we are interested in estimating the mean value to represent the data, we must estimate it based on a frequency distribution.

1. The arithmetic mean of grouped data

To approximate the arithmetic mean of data organised into a frequency distribution, the observations in each class are represented by the midpoints of the class. The mean of a sample of data organised in a frequency distribution is computed by: $\overline{X} = \frac{\Sigma f x}{n}$ where $\overline{X} = \text{mean}$ X = mid value or midpoint of each class, F = frequency in each class Fx = frequency in each midpoints of class. $\Sigma fx = \text{sum}$ of these

Fx = Fx irequency in each imagoritis of class. 2Ix = x sum of these products

n = number of frequencies

Example 6 Given the data group into a frequency distribution below

Monthly Rentals of Halls	No. of units
600-799	3
800-999	7
1000-1199	11
1200-1399	22
1400-1599	49
1600-1799	24
1800-1999	9
2000-2199	4
	120

What is the mean monthly rental for the group data?

Solution

Monthly rentals	No. of units F	X (midpoints)	Fx
of halls			
600-799	3	699.5	2098.5
800-999	7	899.5	6296.5
1000-1199	11	1099.5	12094.5
1200-1399	22	1299.5	28589
1400-1599	40	1499.5	59980
1600-1799	24	1699.5	40788
1800-1999	9	1899.5	17095.5
200-2199	4	2099.5	8398
	120		175340

$$\overline{X}$$
 = $\frac{\Sigma fx}{n}$ = $\frac{175340}{120}$ = 1461.16667

2. The median of grouped data

Recall that the median is defined as the value below which half of the value lays and above which the other half of the value lies. Since the raw data has been organised into a frequency distribution, some of the information is not identifiable. As a result, we cannot determine an exact median. It can be estimated however by:

- i. locating the class in which the median lies
- ii. interpolating within that class to arrive at the median

The rationale for this approach is that the members of the median class are assumed to be evenly spaced throughout the class. The formula is

Median =
$$L + \frac{n}{2} - cfi$$

where, L= lower true limit of the class containing the median N= total number of frequencies, F= frequency of the median class Cf= cumulative no. of frequencies in all classes immediately preceding the class containing the median, I= width of the class in which the median lies

Example 7: Using the frequency in question 6 determine the median

Solution

Monthly rentals	F	Cumulative frequency
600-799	3	3
800-999	7	10
1000-1199	11	21
1200-1399	22	43
1400-1599	40	83
1600-1799	24	107
1800-1999	9	116
2000-2199	4	120

The monthly rentals have already been arranged in ascending order from 600-2199. It is common practice to locate the middle observation by dividing the total number of observations by 2. In this case, $^{n}/_{2} = ^{120}/_{2} = 60$. The class containing the 60th unit is located by referring to the cumulative frequency column in the table above. The 60^{th} rental is in the 1400-1599 class. Recall that the lower limit of the class is really 1399.50 and the upper limit is 1599.50.

To interpolate in the 1399.50 to 1599.50 class, recall that the monthly rentals are assumed to be evenly distributed between the lower and upper true limits, there are 17 rentals between the 43^{rd} and 60^{th} units. And their median is therefore $^{17}/_{40}$. The distance between 1399.50 and 1599.50, is 200, thus $^{17}/_{40}$ of 200 or 85 is added to the lower true limit 1399.50 to give '1484.50' which is the estimated median rental.

To summarise,

Median =
$$1399.50 + \underbrace{60 - 43}_{40} (200)$$

= $1399.50 + \underbrace{17}_{40} (200)$
= $1399.50 + 85$
= 1484.50

Using the median formula, we have

Median =
$$\frac{\text{L1} + \text{n}/2 - \text{CF i}}{\text{F}}$$

= $\frac{1399.50 + \frac{120}{2} - 43 (200)}{40}$
= $\frac{1399.50 + \frac{17}{40} (200)}{40}$
= $1399.50 + 85$
= 1484.50

3. The mode of grouped data

Recall that the mode is defined as the value that occurs most frequently. For data grouped into a frequency distribution, the mode can be approximated by the midpoint of the class containing the largest number of frequencies. Using the table in example 5, the class with the highest number of frequency has the mode. Therefore the mode is 40 in the class of 1400 - 1599.

4.0 CONCLUSION

In this unit you have learnt that the mean, median and the mode are crucial in statistical analysis. It cuts across all disciplines. It is used daily by every individual in daily life be it in the home, office, or business. It is very essential to know the basics of the measures of central tendency as a source of evaluation of many activities in business and other variables of life.

5.0 SUMMARY

In this unit, you are now conversant with the meaning and scope of the mean, median, and mode. Though the scope is wide and inexhaustive, the basic foundational knowledge of the measures of central tendency will help you cope with the challenges of other courses.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. The following data represents the distribution of pure water in Jos Nigeria. 44,56,42,56,76,34,80,37,56,45,46,92,56,40,49,44,68. Find the mean, the mode and the median.
- 2. Find the mean and mode from the following data. 1000, 2000, 4000, 3000, 6000, 4000, 6000, 3000, 9000. List the properties of the median.

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UNIT 4 MEASURES OF DISPERSION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Range
 - 3.2 Average Deviation
 - 3.3 Variance
 - 3.4 Standard Deviation.
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

In this section, you will be exposed to several measures that describe the dispersion, variability or spread of the data. The measures include: the range, average, deviation, variance, standard deviation among others.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the range
- explain variance
- discuss the standard deviation.

3.0 MAIN CONTENT

3.1 RANGE

This is the simplest of the measures of dispersion; it is the difference between the highest and the lowest values in a set of data. Its equation is as follows.

Range = highest value - lowest value

Example 1

The capacities of metal drum container are 38, 20, 37, 64 and 27 litres respectively. What is the range?

Solution. Find the difference between the highest and the lowest numbers in the data. Range = 64 - 20 = 44

Mid range: It is the middle of the range.

[Highest value – lowest value $\div 2$] = 44/2 = 22

SELF-ASSESSMENT EXERCISE

Find the range and the mid range from the following data, 2004, 1988, 2030, 2011, 1981.

3.2 Average Deviation

A serious defect of the range is that it is based only on two values; the highest and the lowest. It does not take into consideration all of the values. The average deviation is often referred to as the mean deviation. It measures the average amount by which the values in the population vary from their mean. In terms of definition, average deviation (A.D.) is the arithmetic mean of the absolute values of the deviation from the arithmetic mean.

In terms of the formula, the A.D. is computed for a sample by

$$A.D = \underline{\Sigma | X - \overline{X}|}$$

Where; X = value of each observation, $\overline{X} = \text{the arithmetic mean of the values}$

n = number of observation in the sample, $\|$ = the absolute value i.e. the signs of the deviations from the mean are disregarded.

Because, we use absolute deviation, the mean deviation is often called the Mean Absolute Deviation (M.A.D.).

Example 2: The height of sample of cartons of sweets is given as; 103, 97, 101, 106, 103. What is the mean deviation? How is it interpreted?

Solution The arithmetic mean weight is 102, found by the sum of all observations divided by 5. To find the A.D., take the following steps.

- i. The mean is subtracted from each value
- ii. The absolute deviations are summed
- iii. The sum of A.D. is divided by the number of values

Weights	$X-\overline{X}$	A.D. $ X-\overline{X} $
103	1	1
97	-5	5
101	-1	1
106	4	4
103	1	1
		$\Sigma \mathbf{x} - \mathbf{x} = 12$

A.D =
$$\frac{\sum |\mathbf{x} - \mathbf{x}|}{\mathbf{n}}$$
=
$$\frac{1^{2}/_{5}}{2.4}$$

The average deviation of the sample is 2.4; the interpretation is that the height of the carton deviates on the average 2.4 from the mean weight of 102.

The A.D. has 2 advantages

- i. It uses the value of every item in a set of data, in its compilation.
- ii. It is easy to understand

However, absolute values are difficult to work with, so the average deviation is not frequently used.

3.3 The Variance

The variance is the arithmetic mean of the squared deviation from the mean.

1. Population variance

The formula for the population variance and a sample variance are slightly different. The population variance is found by;

$$F^2 = \frac{\sum (x-U)^2}{N} \quad \text{or } \frac{\sum x^2 - (\sum x)^2}{N}$$

Where; F2 = population variance, X = the value of the observation in the population, U = the mean of the population, N = total number of observations in the population.

Example 3: The ages of all patients in ward A in Abuja clinic are 38, 26, 13, 41, 22 years. What is the population variance?

Solution

Ages x	x-µ	$(\mathbf{x}\mathbf{-}\boldsymbol{\mu})^2$	\mathbf{X}^2
38 26	10	100	1444
	-2	4	676
13	-15	225	169
41	13	169	1681
22	-6	36	484
$\Sigma x = 140$		$\Sigma(x-\mu)^2 = 534$	$\Sigma x^2 = 4454$

Like the range and A.D., the variance can be used to compare the dispersion in 2 or more sets of observation.

3.4 Sample Standard Deviation

A sample S.|D. is used as an estimator of the population standard deviation. The sample deviation is the square root of the sample variance. It is found by the formula;

$$S = \sqrt{\frac{\sum (x-x)^2}{n-1}}$$

or using the more direct formula the standard deviation can be given as

$$S = \sqrt{\frac{\Sigma x^2 - (\Sigma x)^2}{N}} \sqrt{\frac{n-1}{n-1}}$$

Using the previous example 5, the standard deviation can be determined as the square root of the variance as follows.

$$S = \sqrt{10} = 3.16$$

4.0 CONCLUSION

In this unit, you have learnt that the measures of dispersion determine the rate at which the data spread. It is of importance to know the rate of variability of any given data, this helps us to ensure that there is even distribution in data collection as it is used daily by every individual in life, be it in the home, office, or business. For instance, it is very essential to know the rate of spread of customers, (consumers) for a business man.

5.0 SUMMARY

The unit has thrown more light on the meaning and application of range, deviation and standard deviation. The unit therefore examined the basic concepts of measures of dispersion as a means of launching you to study other units effectively.

6.0 TUTOR-MARKED ASSIGNMENT

1. The followings represent the number of students expelled in a given session 38, 26, 13, 41, 22 years. What is the population variance?

2. The following data represents the performance of some students in some courses: Mr.Abo.56,67,46,80,52,48,68,74. Mrs.Walter 72,43,59,71,50,58,90,44. Determine the range and the midrange in the performance of the two students.

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MODULE 4 STATISTICAL ANALYSIS

Unit 1 Analysis of Correlation Unit 2 Analysis of Regression

UNIT 1 ANALYSIS OF CORRELATION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Correlation Coefficient
 - 3.2 Ranked Correlation
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor- Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

Correlation is a statistical method used to define or establish a relationship that exists between two or more variables. This relationship could be positive, negative or zero. The method of establishing this relationship is known as scatter diagram. In this unit, you will learn how to determine correlation using statistical equations.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the basic concept of correlation
- discuss rank correlation
- explain correlation coefficient.

3.0 MAIN CONTENT

3.1 Correlation Coefficient

Correlation is a statistical method used to define or establish a relationship that exists between 2 or more variables. This relationship could be positive, negative or zero. The method of establishing this relationship is also known as scatter diagram. A scatter diagram measures the closeness of variable among themselves and vice-versa.

The formula for correlation coefficient is as follows.

$$r = \frac{\sum xy}{\sum x^2 \sum y^2}$$

Y	X	Y	$\frac{\mathbf{x}}{X} = X -$	$\overline{Y} = Y - Y$	XY	X^2	Y^2
1	6	30	-6	11	-66	36	21
2	9	15	-3	-4	12	9	16
3	15	16	3	-3	-9	9	9
4	18	15	6	-4	-24	36	16

Calculate the correlation co-efficient for the above data and interpret your result.

$$\vec{X} = 48 = 12, \quad \vec{Y} = 19$$
 $r = \frac{-84}{\sqrt{90 \times 162}} = r = \frac{-84}{120.74} = -0.70$

The above shows that there is negative correlation.

r > 0.5 = Strong correlation

r < 0.5 = Weak correlation

r = 0 – No correlation

If r = 1 there is perfect correlation between the variables

r = negative = There is inverse relationship

SELF-ASSESSMENT EXERCISE

The following data represents the performance of some students in some courses

Mr. Abo.56,67,46,80,52,48,68,74. Mrs. Walter 72,43,59,71,50,58,90,44. Determine the correlation between the performance of the two students.

3.2 Rank Correlation

This method involves ranking variables according to the magnitude of their occurrence without altering the format with which the observations occur in a given data. The ranks that are allocated to each of the observation are used to measure the level of correlation between the variables that occur in that data. The formula for rank correlation is as follows.

$$1 - \frac{6\sum d^2}{n(n^2 - 1)}$$
 Where d = deviation n = No. of observation

Seven students have the following as their scores in GST and Statistics

X	GST	Statistics	Rank (x)	Rank Y	d = x-y	d^2
X_1	70	86	4	1	3	9
X_2	92	71	1	3	-2	4
X_3	89	80	2	2	0	0
X_4	50	63	5	4	1	1
X_5	41	50	6	5	1	1
X_6	82	34	3	6	-3	9
X_7	40	31	7	7	0	0
						24

Compute the rank correlation of students' performances in each subject and explain whether there is any relationship between the two courses.

$$\begin{array}{rcl}
1 - \underline{6(24)} & = 1 - \underline{144} \\
7(49 - 1) & & 336 \\
& & 1 - 0.43 \\
& = 0.57
\end{array}$$

This shows that the correlation is strong since the value of the correlation is greater than 0.5

Ties in rank for correlation, the method of solving the problem remains the same.

	A	В	С	D	Е	F	G	Н	I	J
Rank by X	4	2	8	4	7	6	10	1	3	9
Rank by Y	3	1	5	8	5	9	5	2	4	10
Rx	4.5	2	8	4.5	7	6	10	1	3	9
Ry	3	1	6	8	6	9	6	2	4	10
D	1.5	1	2	-3.5	1	-3	4	-1	-1	-1
D^2	2.25	1	4	12.25	1	9	16	1	1	1

$$\sum d^{2} = 48.50$$

$$R = 1 - \frac{6\sum d^{2}}{10(100 - 1)}$$

$$= 1 - \frac{6 \times 48.5}{99}$$

$$= 1 - \frac{9.70}{33}$$

$$1 - 0.29$$

$$= 0.71$$

SELF-ASSESSMENT EXERCISE

The following are the performances of some contractors of some projects completion:

Contractor X's performance: 82%, 57%, 74%, 40%, 52%, 51%. Contractor Y's performance: 71%, 34%, 50%, 81%, 62%, 54%. Calculate the rank correlation.

4.0 CONCLUSION

You have learnt that correlation is a statistical method that is used daily by people in transaction and business, as long as it involves comparison. It cuts across all disciplines and it is used in daily life by every individual in the home, office, or business. It is very essential to know the basics of correlation as a means of evaluation of any transaction that involves comparative analysis.

5.0 SUMMARY

The unit has thrown more light on the meaning and scope of correlation. Even though the scope is wide, the basic foundational knowledge of correlation will help you cope with the challenges in business as managers. The unit therefore examined the basic concepts of correlation, correlation coefficient, rank correlation.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. The following data shows the attendance of lectures and the rate of passing the examination in a given course.

 Attendance: 20 19, 34, 25, 18 and 22. Pass rate: 170, 170, 230, 200, 180 and 190. Calculate the correlation between attendance and the pass rate.
- 2. Seven dealers have the following as their scores in the distribution of Gt and St

W	Gt	St
\mathbf{w}_1	70	86
\mathbf{w}_2	92	71
\mathbf{w}_3	89	80
W_4	50	63
W_5	41	50
W_6	82	34
\mathbf{W}_7	40	31

a) Compute the rank correlation of the distributors in product Gt and St and explain whether there is any relationship between the two products.

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UNIT 2 ANALYSIS OF REGRESSION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Regression Analysis
 - 3.2 Estimate Regression Equation
 - 3.3 Forecast using Regression Equation.
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor- Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

Regression is a statistical method of determining the best fit. It is used to measure the rate of dispersion along any given curve.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the concept of regression
- estimate regression equation
- forecast by using regression equation.

3.0 MAIN CONTENT

3.1 Analysis of Regression

Regression is a quantitative ways of arriving at the best fit. It is also referred to as the line of average in any given data.

Example 1:

The following data represents sales from a particular company as a result of advertising its product.

Sales(Y)	Advert(X)	x=X-X	$Y = Y - \overline{Y}$	Xy	X^2
82	20	0	2	0	0
70	46	-4	-10	40	16
90	24	4	10	40	16
85	22	2	5	10	4
73	18	-2	-7	14	4
= 400	= 100			104	40

$$\overline{Y} = \underline{400} \qquad \overline{X} = \underline{100} \\ 5 \qquad \overline{5}$$

$$\overline{Y} = 80$$
 $\overline{X} = 200$

Attempt a regression of Y on X

$$Y = a + bx$$

$$b = \frac{\sum [(x - \overline{X}) (y - \overline{X})]}{\sum (x - \overline{X})^2} = \frac{\sum xy}{\sum x^2}$$

$$b = \frac{104}{40} = 2.6$$

$$a = \overline{Y} - b\overline{X}$$

$$a = 80 - (26) (20)$$

$$a = 80 - 52$$

$$a = 28$$

$$\overline{Y} = 28 + 2.6 \hat{x}$$

There is a positive relationship between x and y, that is, as x increases y also increases. It means advert leads to increase in sales.

3.2 Estimate the Regression Equation.

Example 2:

An employer of labour wants to find the relationship between the labour input employed and the total output using the following hypothetical data.

Labour input	Output
0.8	28
1.1	31
1.6	29
2.3	20
2.2	37
3.1	35
3.0	40
4.6	56

- (a) Establish the least square regression for the above data
- (b) Using the average in the data, sketch the regression line.
- (c) Assuming the labourer decides to employ 80 workers, estimate the output for his firm. Y on X.

Output(Y)	Input(X)	XY	X^2
28	0.8	22.4	0.64
31	1.1	34.1	1.21
29	1.6	46.4	2.56
20	2.3	46.0	5.29
37	2.2	81.4	4.84
35	3.1	108.5	9.61
40	3.0	120.0	9.00
56	4.6	257.6	21.16
= 276	= 18.7	= 716.4	= 54.31

$$Y = mx + c$$

$$Y = a,x + ao$$

$$Y = bx + a$$

$$Y = \sum Y = \frac{276}{8}$$

$$X = \sum X$$

$$N$$

$$= 2.34$$

Last square equation

$$nc + m\sum x = \sum y$$

$$c\sum x + m\sum x^2 = \sum xy$$

where

Let's substitute 6.72 for m in equation (1)

8C + 18.7 (6.72) = 276.0

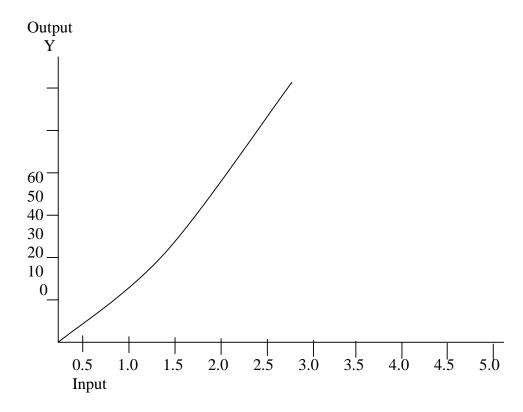
8C + 125.66 = 276

8C = 276 - 125.66

8C = 150.34

C = 18.78

Y = 6.72X + 18.79



3.3 Forecasting using Regression

Using the question in 3.2 above we can forecast based on the estimated regression equation.

Assuming the labourer decided to employ 80 workers when the input =

Y = 6.72(80) + 18.79

Y = 537.6 + 18.79

Y = 556.39

4.0 CONCLUSION

In this unit you have learnt that regression is one the fundamental methods of estimating any statistical equation. It cuts across all disciplines and it is also used in forecasting in business. Therefore a good knowledge of regression can help a manager to forecast sales, market performance etc.

5.0 SUMMARY

The unit has thrown light on the meaning and scope of regression. The scope and application are wide and the basic foundational knowledge of the unit will help you cope with the challenges in business management and forecasting. The unit therefore examined the basic concepts of best fit as a means of graphical exposition.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. The following data shows the death and the birth rate in a given city.
 - Death rate: 20, 19, 34, 25, 18 and 22. Birth rate: 170, 170, 230, 200, 180 and 190. Estimate the regression equation for death and the birth rate of the city.
- 2. Forecast the birth when the death rate is 58, 66 and 80.

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MODULE 1

Unit 1	The Project Cycle
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Unit 3	Factors Affecting the Location of Projects
Unit 4	Capacity and Production Planning
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UNIT 1 THE PROJECT CYCLE

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 The Project Cycle Meaning and Stages
 - 3.1.1 The Project Idea Stage
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1.0 INTRODUCTION

Ordinarily, a project is an undertaking that requires concentrated effort. Projects require commitment and deployment of resources. As a business manager of the future, you will encounter various projects in your work place or environment. Most of the projects are likely to be private sector driven. They may be manufacturing projects or they could be petrochemical or civil engineering projects. Your key task, as a project evaluator, is to carefully consider each and every project brought to your attention and see how <u>useful</u> or <u>valuable</u> they are.

Our first task and which we will accomplish in this unit is to examine the concept of a project cycle. This concept is very important as it gives us an overview of projects. The knowledge so gained, will lead us throughout the duration of this course.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain a project cycle
- describe the sequences in a project cycle.

3.0 MAIN CONTENT

3.1 The Project Cycle – Meaning and Stages

A project cycle tries to describe the various stages that are involved, from the conception of a project idea to when the project is executed or actually takes off. Understanding a project cycle is very important as it enables us to get the total picture of a project. We will now examine the various stages of a project cycle.

Basically, projects consist of the projection of ideas and activities into new endeavours. As earlier discussed, projects may be public sector projects in agriculture, defence or transportation. A major railway link from Ibadan to Onitsha may constitute a major public sector project. A new brewery springing up at Enugu is a typical example of a private sector initiative. But whatever type of project that we are considering, we need to first understand what is usually known as the project cycle. So before we go into the real subject of our discussion – Project Evaluation, we need to build a strong background. This background is in the form of proper understanding of how a project is conceived. It explains who conceives a project. It also explains the flow of activities up to the execution of the project.

The Project Cycle

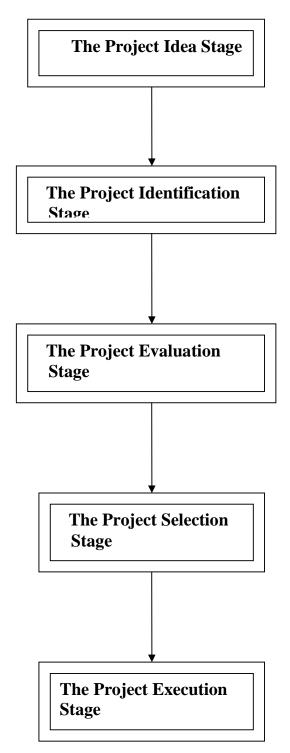


Figure 1 The project cycle

SELF ASSESSMENT EXERCISE 1

List the stages of a project cycle.

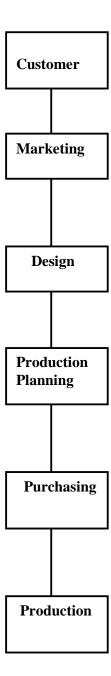


Figure 2 A manufacturing project cycle

3.1.1 The Project Idea Stage

The project idea stage is the first stage of a project cycle. The idea about a project arises from a variety of sources within the internal environment or market place.

New project ideas could originate from within an organisation or from outside the organisation. If the idea originates from within, it could be from a sales person who has encountered some success or problem with customers while performing his or her functions.

You will also realise that a new project idea could emanate from outside an organisation. Coming from outside an organisation, it could be requests from existing customers asking for bigger or better products. New project ideas may fall into any of the following categories.

- Proposal to add new products to existing lines: A company with existing product lines may decide to add new products to its existing lines.
- Proposal to expand capacity in existing lines: A company may have a proposal to expand capacity to enable it take advantage of enlarged market opportunities.

We need to stress that new project ideas may originate from any level in an organisation. A factory cleaner within an organisation may come up with a new product idea. Also an executive director in an organisation may also generate a new project idea.

3.1.2 The Project Identification Stage

After the project idea stage, the next stage is the project identification stage. The project identification stage consolidates the idea stage. Project ideas are not really useful unless they are clearly identified and put down in a systematic manner for further processing.

The idea to introduce a new product into the market may come from a company salesman who is very familiar with the market. At the boardroom room level, the entire organisation has to see the project idea properly and clearly identify it as a possible area of business investment. The totality of the new idea would be considered.

3.1.3 The Project Evaluation Stage

When a project has been identified, the next step is to evaluate the project. Project evaluation involves the estimation of the benefits and costs of a project. Benefits and costs should be measured in terms of cash flows.

We have to emphasise at this point that the estimation of the cash flow of a project is a very difficult task. It is difficult in the sense that the cash flow to be estimated is future cash flow. For example in the year 2007, we will tried to estimate the cash flow for the year 2008.

In a corporate setup, the evaluation of projects should be carried out by a team of experts drawn from the various departments like production, marketing, accounts and administration. The team of experts should be objective in their evaluation of projects.

Alternatively, the evaluation of a project may be contracted to a third party like consultants. Contracting evaluation of projects to outside parties tends to eliminate bias.

3.1.4 The Project Selection Stage

After the project evaluation stage, the next stage is the project selection stage. Faced with an array of projects with different values and worth, there is need to select which projects to embarked upon. There is no standard procedure for selecting projects as this will differ between firms. The important thing to note is that the project selection function is a top management responsibility which in most cases goes to the Board of Directors of an organisation. In selecting projects, management usually considers the financial outlays involved and matches them with the financial capabilities of the firm. For example, a firm that has only N10,000,000 (ten million naira only) investment funds cannot be considering a new investment that involves a capital outlay of N40,000,000 (forty million naira only) except if it can source money externally e.g., from banks.

3.1.5 The Project Execution Stage

The project execution stage is the final stage in the project cycle. After a project has been selected, it moves on to the execution stage. In most organisations, the responsibility for execution of projects is vested on a project management team raised by top management. The function of the team is to ensure that the budget for the project is spent entirely on the project and that the project is completed on schedule.

In an ideal organisation, the project management team usually prepares a monthly budget consideration report on projects for top management consideration. This is important for project monitoring and control.

SELF ASSESSMENT EXERCISE 2

Discuss three sources of new project ideas.

4.0 CONCLUSION

This unit has treated the concept of the project cycle which is a stepping stone into our study of project evaluation. Now that we have built the necessary background, we shall be discussing Project Analysis – an introductory frame work in the next unit.

5.0 SUMMARY

In this unit we have discussed the concept of the project cycle. We have seen that it starts from the project idea stage, goes to the identification stage, to the evaluation stage. From the evaluation stage it moves to the selection stage and finally to the project execution stage.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. Explain what you understand by the term "project cycle".
- 2. List and discuss the various stages involved in a project cycle.

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe, (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 2 PROJECT EVALUATION – AN INTRODUCTORY FORMAT

CONTENTS

- 1.0 Introduction
- 2.0 Objective
- 3.0 Main Content
 - 3.1 Project Evaluation An Introductory Format
 - 3.1.1 The Technical and Engineering Segment
 - 3.1.2 The Management Segment
 - 3.1.3 The Demand and Market Segment
 - 3.1.4 The Financial Segment
 - 3.1.5 The Economic Segment
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In the first unit, we discussed the concept of the project cycle which is very crucial to our understanding of project evaluation. In this unit, we will discuss project evaluation in a proper context. In doing this, we shall build an introductory format which will assist us in our discussion. Evaluation of a project involves a careful consideration of the totality of the project with a view to seeing how useful or valuable it is. Evaluation enables us to attach proper financial value to a project and also allows us the liberty of comparing it with other projects.

You will note that an analysis is not done in a vacuum. It is usually documented.

A problem usually encountered in project evaluation is how to arrange the work to make it readable or understandable.

A very simple format which we will adopt in the evaluation of projects is one that recognises the various functional aspects or units of an organisation.

2.0 OBJECTIVE

At the end of this unit, you should be able to:

• explain the format for project evaluation.

3.0 MAIN CONTENT

3.1 Project Evaluation – An Introductory Format

The introductory format for the evaluation of projects is the arrangement of our work into carefully defined segments. We will now discuss the various segments of project evaluation.

3.1.1 The Technical and Engineering Segment

The technical and engineering segment of project evaluation tries to evaluate the total technical and engineering soundness of a project. It also tries to relate the project to the environment in which it is located.

We will now proceed to draw up a checklist for the technical and engineering segment of project evaluation.

Table 1: A Checklist for the Technical and Engineering Segment

Item	Have you covered this in your evaluation?
Land for the project	
Civil works and foundations	
Steel structures	
Factory building	
Offices	
Wind force and direction	
Availability of electric power	
Availability of generator	
Potable water	
Water bore hole with fittings	
Sewage	
Existing roads	
Access difficulties	
List of machinery / equipment	
Availability of raw materials	
Availability of professionals	
(labour)	
Availability of skilled labour	
Availability of unskilled labour	

3.1.2 The Management Segment

After evaluating the technical and engineering segments of projects, the next segment we need to discuss is the management segment. The management segment focuses attention on the management aspects of a project. Projects only become successful if they are well managed.

We do not need to over-stress the importance of management. Again, we need to evaluate the legal form of the organisation that is being evaluated and see if it can carry the project in question.

Table 2: A Checklist for the Management Segment

Item	Have you covered this in your evaluation?
The legal form of the business	
Organisational structure	
Who will manage the project	
Qualifications of key staff	
Number of employees	
Salaries	

3.1.3 The Demand and Market Segment

The next segment we shall consider is the demand and market segment. This segment focuses attention on the demand for goods and services and relates it to the market. An evaluation of the demand for goods and services is very important because demand translates to revenues. Also, we need to evaluate supply situations in the market. These two topics will be treated in detail later.

We will now examine a checklist for the demand and market segment.

Table 3: A Checklist for the Demand and Market Segment

Item	Have you covered this in your evaluation?
What is the population of the market?	
What is the nature of demand for the	
product?	
What are the factors affecting	
demand?	
What is the size of the market?	
What is the supply situation?	
What is the price of the demand?	
Market share	
Advertisement strategy	
Promotional strategy	

SELF ASSESSMENT EXERCISE

List and explain five items that you hope to find in the checklist of the management segment of a project evaluation.

3.1.4 The Financial Segment

The financial segment of project evaluation focuses attention on the financial aspects of projects.

In discussing financial issues, we are considering all financial aspects of a project such as start-up costs, financial plans, renames and costs and income statements.

Table 4: A Checklist for the Financial Segment

Item	Have you covered this in your evaluation?
Cost of land and buildings	
Cost of utilities	
Cost of machinery /equipment	
Revenues	
Expenditures	
Projected income statement	
Projected cash flow statement	
Projected balance sheet	

3.1.5 The Economic Segment

The last segment we will consider is the economic segment. The economic segment considers projects from the macro economic point of view. Economic analysis tries to measure the benefits and costs of projects in terms of their value to society as a whole.

Table 5: A Checklist for the Economic Segment

Item	Have you covered this in your evaluation?
Employment generating capacity	
Contribution to economic growth	
Contribution to government	
revenue	
Forward linkages of the project	
Backward linkages	
Externalities	

4.0 CONCLUSION

What we have achieved in this unit is to develop a format for conducting the evaluation of projects.

5.0 SUMMARY

We have discussed the format of project evaluation. We did identify the following as segments of project evaluation.

- The technical and engineering segment
- The management segment
- The demand and market segment
- The financial segment
- The economic segment

6.0 TUTOR-MARKED ASSIGNMENT

Discuss the key segments of project evalution

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 3 FACTORS AFFECTING THE LOCATION OF PROJECTS

CONTENTS

- 1.0 Introduction
- 2.0 Objective
- 3.0 Main Content
 - 3.1 Factors Affecting the Location of Projects
 - 3.1.1 Nearness to Markets
 - 3.1.2 Nearness to Sources of Raw Materials
 - 3.1.3 Nearness to Power Supply
 - 3.1.4 Nearness to Water Supply
 - 3.1.5 Availability of Good Road Networks
 - 3.1.6 Availability of a Good Rail System
 - 3.1.7 Nearness to Airports
 - 3.1.8 Availability of Land
 - 3.1.9 Political Considerations
 - 3.1.10 Intervention Projects
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 2 we discussed the format for the evaluation of projects. In this unit, we shall discuss factors affecting the location of projects. This is important because it enables us as analysts to have a proper knowledge of the key issues relating to project locations. Understanding this unit will give you the advantage of knowing in advance what should be and what should not be when issues of project location are raised. For example if a project is located in an area without any justification, the project analyst should be aware of this based on his or her previous knowledge of project location issues.

2.0 OBJECTIVE

At the end of this unit, you should be able to:

• explain the factors affecting the location of projects.

3.0 MAIN CONTENT

3.1 Factors Affecting the Location of Projects

Whether in the private sector or the public sector, projects are located according to set guidelines. In the private sector, project location is based purely on economic guidelines, usually put in place by the various organisations. These guidelines regulate the ways and means by which projects are located. In the public sector, projects may be located according to a different set of guidelines and they may not be driven by profit motive. It is the discussion of these factors affecting the location of projects that will now take our time.

3.1.1 Nearness to Markets

A major factor affecting the location of projects is their nearness to markets. Most businesses are established primarily for profit.

A business organisation will make profit if the goods or services it produces are purchased by third parties. It is the difference between revenues and costs that constitute profit. You will easily and quickly realise that most populations that constitute markets are located mainly in the urban areas of Nigeria. So we shall hold firmly to the fact that the nearness to markets is a major factor influencing the location of most projects in Nigeria. Most of the entire population of Nigeria is located in such areas like Lagos, Onitsha, Port Harcout, Kano, Ibadan, Abuja, etc. Because of this, most businesses and government establishments are located near the critical markets that they wish to serve. That is to say that businesses follow their markets. Concentrations of people or critical markets are very important to corporate officials who design marketing plans.

If you take a good look at your environment, you will realise that a lot of companies are located within Lagos. It is not by accident. Rather it is by design. Businesses are attracted to areas of core concentration of people who constitute vital markets.

3.1.2 Nearness to Sources of Raw Materials

Nearness to sources of raw materials is another strong factor affecting the location of projects, especially manufacturing concerns. Service based projects can locate without reference to any raw material source. But manufacturing concerns cannot afford to gloss over the issue of raw materials.

For a manufacturing concern, the raw material source is very important because in most cases, raw materials may constitute over 80% of a product. If a company decides to establish its factory very far away from its raw material source, it is going to end up spending a lot of money to transport raw materials from the source to its factory. This will mean added costs and in the long run, the company may not be able to compete effectively in its chosen market.

If you sit back and think deeply, you will see patterns of industrial locations that tend to suggest that most industrial projects are located close to the sources of raw materials. The Nigerian Coal Corporation Enugu is located in Enugu close to the source of coal.

Most cocoa processing firms are located in the western parts of Nigeria because cocoa is found in abundant quantities there.

In the eastern parts of the country, oil mills and vegetable oil plants litter the economic landscape simply because the oil palm grows wildly there and it is also the major raw material for palm oil manufacture and vegetable oil.

If you go up north, you will also realise that the tea producing company is located at the Mambilla Plateau because the plateau is very rich in tea cultivation.

3.1.3 Nearness to Power Supply

Power supply is a very critical input in business, whether service based or manufacturing based. In the case of service-based concerns like banks, electricity will be needed to power computers, servers and other equipment needed to provide service to customers.

In a manufacturing concern, electricity is everything. Without electricity, a manufacturing concern is not likely to operate. Electricity is needed to mix chemicals, to drive machinery and equipment and to start electric motors etc.

So in the choice of where to locate, every concerned establishment or business should properly address the issue of power supply.

Interruptions in power supply create production problems for industries; this also translates to inability to meet production schedules, and inability to meet production schedules leads to frequent customer complaints for failed deliveries. Frequent failures to deliver goods on schedule may lead to loss of customers.

In our peculiar Nigerian situation, irregular power supply has taken its toll on the manufacturing sector and even service-based activities. The result of this poor public power supply has led most organisations to operate their own plants but with the attendant high operating costs of both fuel and servicing of generating plants.

3.1.4 Nearness to Water Supply

Water is very important in business especially in manufacturing industries. Many manufacturing companies use a lot of water for manufacturing activities.

Soft drink manufacturers for example, use a lot of water in the production process.

In the agricultural sector, garri processing factories use a lot of water and they are attracted to areas where there is plentiful supply of water.

3.15 Availability of Good Road Networks

Availability of good road networks is another major factor influencing the location of projects in the economic landscape. Road networks are very important. They are important for the movement of essential raw materials from raw material sources to factories and also for the movement of finished goods to the markets where they are needed.

Most investors in the economy are usually attracted to areas with good road networks. Good road net works reduce the cost of transportation.

3.1.6 Availability of a Good Rail System

Another important factor influencing the location of projects is the availability of a good rail system. A good railway system ensures cheap transportation and evacuation of raw materials from their sources to factory locations and also the movement of finished goods to markets.

You may observe that the development of trading locations in Nigeria seemed to have followed the railway system. The North – Eastern railway system runs through towns today which have become trading posts. Aba, Umuahia, etc., all enjoy good trading activities because they are located along railway line routes.

3.1.7 Nearness to Airports

Another identified factor affecting location of projects is nearness to airports. A lot of businesses tend to be located close to airports. Airports encourage quick movement of people to and from various locations. If you take a good look at the country today, the towns that are served with air links tend to be enjoying faster economic growth and development. Port Harcourt, Calabar, Enugu, etc., enjoy good air links which facilitate the movement of people.

3.1.8 Availability of Land

In economic theory, there are four factors of production namely: land, labour, capital and the entrepreneur. Each of these factors is very important. However, Land appears to assume very important dimensions especially where large scale agricultural projects are involved. Very large agricultural projects like oil palm plantations require vast areas of land before they can become profitable. A cassava farm which will feed a garri processing plant needs to be very vast in terms of land space. You may realise that invariably projects are attracted to areas where there is adequate land.

3.1.9 Political Considerations

The location of most business projects is driven mainly by economic motives. Private sector projects are mainly profit-driven and their location is based only on economic merits. Also the public sector, since the era of economic reforms, has bought the idea of economic reforms and is now locating projects based on economic merit and viability.

However, not all projects are located based on sound economic judgment. Political considerations occasionally play very important roles in deciding where a project will be located. For example an oil refinery may be located very far away from crude oil sources. The cost of transporting crude oil to the refinery may result in the refinery operating at a loss.

3.1.10 Intervention Projects

Intervention projects are those projects which are conceived and located within specific areas to correct inequalities in distribution of resources. In the emerging political dispensation, the issue of uneven development has been brought to the front line of discussions. Complicating the discussions is the issue of resource control and the attendant political and social implications.

The Niger Delta region of Nigeria produces a major percentage of the oil revenues of Nigeria. Recent thinking is that the region has not received sufficient attention as a major oil producing region. Currently, the federal government is focusing attention on the region and a lot of developmental projects are now springing up in the area.

SELF ASSESSMENT EXERCISE

List and discuss four factors that have influenced the location of industrial projects in your present environment.

4.0 CONCLUSION

In this unit, we have discussed those factors which affect the location of projects in Nigeria. The projects may be private or public sector projects. The factors may be economic or in some cases be political.

5.0 SUMMARY

In this unit, we have discussed those factors that affect the location of projects in Nigeria. They range from market driven forces to political forces. We have also discussed intervention projects which are a new feature in our economy.

6.0 TUTOR-MARKED ASSIGNMENT

- a. Mention four factors that are likely to influence the location of projects in Nigeria.
- b. Discuss the four factors that you have mentioned.

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 4 CAPACITY AND PRODUCTION PLANNING

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Capacity and Production Planning
 - 3.2 Concepts of Capacity
 - 3.3 The Nature of the Production Plan
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 70. References/Further Readings

1.0 INTRODUCTION

In Unit 3, we discussed factors affecting location of projects. There we examined such factors as nearness to critical markets, nearness to sources of power and other factors. In this unit, we shall discuss capacity and production planning.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain capacity and production planning
- describe the practical applications in industry.

3.0 MAIN CONTENT

3.1 Capacity and Production Planning

Our interest in capacity and production planning is for two main reasons. First, it could be that we are project initiators trying to draw up a business plan for a start-up project in which case we want to know well in advance what the capacities of our plants will be and the production plan. Understanding capacity of a project assists us to design and construct such items like cash flow statements, etc. On the other hand, we could be analysts on the other side of the fence. In this case, we must have been presented with a business plan or a project plan and have been asked to evaluate the project. So no matter on which side we are, we must understand capacity and production plans. No firm undertakes to produce goods without production plans. Production plans are linked to the firm's demand for goods and services. We shall now

discuss "capacity" in relation to production plans. The production plans are in relation to the manufacture of tangible goods.

3.2 Concepts of Capacity

Ordinarily, capacity means "ability to do something". But in project terms, the capacity of a plant is a reference to the output of the plant or machine. When discussing plant capacity, we should realise that there are three types of capacity namely:

- Installed capacity
- Attainable capacity
- Actual capacity

Installed Capacity

The installed capacity of a plant or machine is the maximum output in terms of tangible goods which the plant or machine is capable of producing at a given time. The capacity is usually given by the manufacturers of the equipment.

We can therefore define installed capacity in terms of the following:

- Number of goods produced per minute
- Number of goods produced per hour
- Number of goods produced per day
- Number of goods produced per year

In practice, we usually define installed capacity in terms of annual capacity. The best way to visualise installed capacity is to consider a Mercedes Benz car that has a speedometer limit of 200km per hour. In effect, 200km/hour is the maximum speed the car can attain according to the manufacturers. This 200km per hour is the installed capacity of the car.

SELF ASSESSMENT EXERCISE 1

A plant can produce 2000 sachets of water per hour, all things being equal. What is the installed capacity of the plant per annum?

Attainable Capacity

We have just discussed the installed capacity of a plant or machine and described it as the maximum output that a plant or machine can achieve. However in real life, no machine can work for 60 minutes in an hour, 24 hours a day and 365 days in a year without breaking down. There must

be production loss arising from servicing of the machines, changes in shift, repairs of machines, etc.

Because of these reasons, we now have a more realistic capacity which in the literature is known as the attainable capacity of a plant/machinery. The attainable capacity captures the effect of plant servicing and or repairs on production output.

Let us go back to our Mercedes Benz car. We said it has an installed capacity of 200km per hour. Practically, we know that no person will like to drive a car at a speed of 200km per hour. Everyone should be thinking of safety. We could agree amongst ourselves that 100km per hour constitutes a safe speed. In that line of thinking, we can now define 100km per hour as the attainable capacity of the car. This is the speed at which a sane person can drive the car.

Actual Capacity

Actual capacity refers to the real output that a plant can achieve given the market situation. We should realise that the actual capacity of a plant depends on a lot of factors. First is the issue of the market for the plant's output of goods. A plant should produce only what it can sell at a given time.

Again financial resources can limit the output of a plant because firms require finance to purchase raw materials and other inputs for production.

3.3 The Nature of the Production Plan

The production plan shows exactly the proposed output of a plant or project under consideration. In constructing the production plan, information on the plant capacity, shift arrangements, raw material sourcing, etc., are all used. Below is the production plan of a vegetable oil refining plant that uses palm kernel oil as base raw material.

You are requested to properly study the production plan and make sure you understand it.

An Example of a Production Plan

The Plant

The plant under consideration is a modern vegetable oil refining plant using palm kernel oil (PKO) as base raw material. The installed capacity of the plant is 50 metric tons per day of 24 hours.

Working on a shift of 10 hours, an output of 20.83 tons is possible, while two (2) shifts of 10 hours each will yield daily output of 41.66 tons of refined vegetable oil. Because of the need to accomodate plant repairs, servicing, public holidays, etc., the plant will be operated for 300 working days in a year. Proposed capacity is based on 2 shifts of 10 hours each per day.

To ensure that there is uninterrupted supply of palm kernel oil to feed the plant, the project promoter already has in operation 26 Muar ban Lee Palm Kernel Oil (PKO) expellers with a total Installed capacity of 220 tons/day.

Table 6: Annual Capacity/Production Plan (Metric Tons)

	Year 1	Year 2	Year 3	<u>Year 4 – 5</u>
	2007	2008	2009	2010- 2011
Installed Capacity	18,250	18,250	18,250	18,250
Attainable Capacity	16,425	16,425	16,425	16,425
Proposed Capacity	12,498	12,498	12,498	12,498

Proposed capacity is based on the following:

Year 1 to 5: 2 shifts of 10 hours each per day and 300 working days/year.

Attainable capacity is 90% of installed capacity.

Installed capacity: Installed capacity is 50 tons per day x 365 days. This translates to 18,250 tons per annum

Production Activities

STAGE ONE Palm Kernel Crushing

Stage one of production involves the crushing of palm kernel to yield palm kernel oil (PKO). The residues are palm kernel cake (PKC) and palm kernel sludge.

Palm kernel cake is sold to feed mills as a major ingredient for animal feed. Palm kernel sludge is sold to soap manufacturers.

Refining Of Oil to Yield Edible Vegetable

STAGE TWO

Stage two of production involves the refining of the crude Palm Kernel Oil (PKO) to yield edible vegetable oil.

In refining of the oil, fatty acid is collected. The fatty acid is sold to soap manufacturers.

Activity Flow

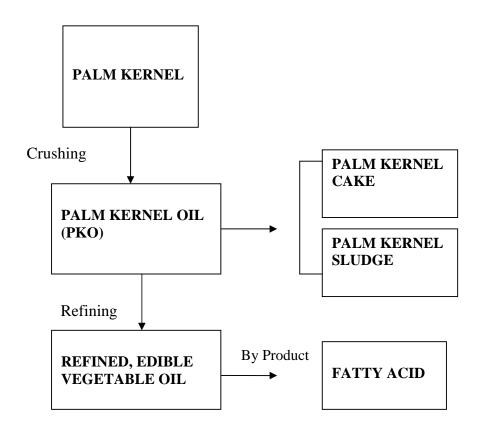


Figure 3: Activity flow in a vegetable oil refining plant

Machine and Equipment Required

The vegetable oil refining plant consists of the following equipment broken down into various sections:

- 1. Continuous oil pre-treatment section
- 2. Continuous bleaching section
- 3. Continuous physical refining and deodorising section

- 4. Thermal oil heating unit
- 5. Water cooling and recirculation system
- 6. Steam generation and distribution system

These machinery and equipment will be sourced from overseas. However, in addition, the following items form part of the equipment required but they will be sourced locally:

- 1. 2 units crude oil tank 200 tons
- 2 1 unit refined oil tank 300 tons
- 3. 1 unit fatty acid tank 50 tons
- 4. 1 unit furnace oil tank.

Vegetable Oil Refining

Process Description

Pre Treatment

The crude oil received in the crude oil tanks, if required, is given a hot water treatment and allowed to settle to remove settled impurities along with water. The oil is then heated till drying and transferred to high shear mixer wherein the requisite quantity of phosphoric acid is dosed in to allow conversion of non hydratable phosphotides. The acidulated mixture is then given a retention time for complete reaction of phosphoric acid in the reaction vessel.

Bleaching

The heated oil (acid conditioned, degummed or neutralised) enters the inlet mixing tube of the vacuum bleacher. Bleaching earth is proportioned by a valve dosing system that also deaerates the earth before it enters the vessel. The earth is introduced under the oil spray in the mixing tube for improved mixing, so that earth does not enter the bleaching vacuum system. The earth absorbs any residual gums and soap as well as trace metals, colour bodies and other polar impurities. The bleacher has multiple agitated compartments and operates under vacuum to ensure complete moisture and air removal. The dried oil and earth mixture is discharged by the bleacher discharge pump to one of the filters where the spent earth is removed. The filtered oil is collected in the bleached oil receiver. The bleached oil pump sends the oil for deodorizing via one of the alternating polish filters.

Filter Management

The standard bleaching system is based on the use of two filters of which one is on line while the other is being cleaned and prepared for the next cycle. When a filter has reached maximum capacity, as indicated by high feed pressure, it is taken offline for cleaning and replaced by the other filter.

The cleaning procedure begins by first pressurising the filters with steam to expel residual oil into the Heel Tank and dry the spent filter cake. The oil recovered from the filter cake is collected in the tank and can usually be recycled to the bleacher. After drying, a bottom discharge valve on the filter is opened and the leaves pneumatically vibrated so that the spent cake is dislodged and discharged via a hopper. The filter is then closed and prepared for the next batch. Before going online again it can be pre-coated with filter aid and/or bleaching earth in order to seal the filter.

Raw Material Requirements

The key raw material required for the refining of vegetable oil is palm kernel oil (PKO). The rest are additives.

Table 7: Raw Material Requirement

Material	Sources	Unit Cost
Palm kernel oil	Open markets in Onitsha, Owerri	N130,000/Ton
Citric acid	Pharmaceutical firms, open markets at Aba, Port-Harcourt, Owerri and Lagos.	N200/kg
Phosphoric acid	Freely purchased in chemical shops at Aba and other locations nationwide.	N300/kg
Bleaching earth	1) Vin George Ventures 7 Association Avenue, Lagos 2) Open markets at Aba, PH, Onitsha etc.	N88,000 per Ton

Annual Input-Output Analysis

The annual input–output analysis seeks to determine the inflow of manufacturing raw materials and their corresponding outflow of products. It assumes 300 working days per annum. It aggregates the daily input-output analysis.

Table 8: Annual Input – Output Analysis

Input	Output
32,889.60 tons of Palm Kernel	(1) 13,155.84 tons of Palm Kernel Oil (PKO)
	(2) 18,418 tons of Palm Kernel Cake (PKC)
	(3) 1,315 tons of Palm Kernel Sludge.
13,155.84 tons Palm Kernel Oil	(1) 12,498 tons of Vegetable Oil
(PKO)	(2) 657.84 tons of fatty acid

Table 9: Summary of the Proposed Production Plan (Tons)

Year Ended 31st Dec

		Year 1	Year 2	Year 3	Year 4-5
1.	Refined	2007	2008	2009	2010-2011
	vegetable oil				
2.	Palm kernel cake (PKC)	12,498	12,498	12,498	12,498
		18,418	18,418	18,418	18,418
3.	Palm kernel sludge	1,315	1,315	1,315	1,315
4.	Fatty acid				
		657.84	657.84	657.84	657.84

4.0 CONCLUSION

We have discussed capacity and production planning which are very important aspects of a project because they both relate the project to the market. Capacity and production plans enable the firm to plan well in advance what to produce and in what quantity too.

5.0 SUMMARY

We have discussed capacity and production planning and have established the link between them. We also used an example of a vegetable oil refining plant to explain the production plan.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. What do you understand by the installed capacity of a plant?
- 2. How is it different from the attainable capacity?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999): *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 5 THE CONCEPT OF ENGNEERING EVALUATION

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 The Concept of Engineering Evaluation
 3.1.1 Identification of Plant and Equipment
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 4, we discussed capacity and production planning. There we discussed installed capacity, attainable capacity and actual capacity. In this unit, we shall discuss the concept of engineering evaluation.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the concept of engineering evaluation
- discuss engineering evaluation.

3.0 MAIN CONTENT

3.1 The Concept of Engineering Evaluation

A project consists of many sub-segments depending on its nature. There may be mining projects or manufacturing projects. As we discussed earlier, engineering is a major segment of many projects. Also projects have their management segment, marketing segment, etc. The purpose of engineering evaluation is to evaluate and break-down in simple terminology the engineering feasibility of a project. For example, a project may be a manufacturing project.

The type of questions we need to ask will involve the manufacturing processes and the equipment that are involved. In a manufacturing project, engineering evaluation will entail the following:

- 1. Identification of plant and equipment
- 2. Identification of the necessary raw materials
- 3. The production process.

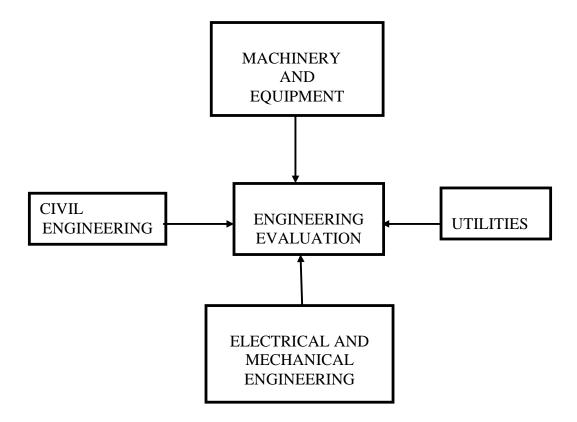


Figure 4: Components of engineering evaluation

3.1.1 Identification of Plant and Equipment

In a manufacturing project, the most important element, apart from land and buildings is the plant and machinery that will be used for manufacturing.

Plant and machinery are usually supplied by the manufacturers. In practical terms, when a manufacturer (vendor) is supplying a manufacturing or equipment, the vendor should give sufficient details of the machine.

The details of the machine will assist the project evaluator. All projects do not have the same layout and features and it is the duty of the project evaluator to segregate the project under study and see whether it is feasible or not from an engineering perspective.

Technical Support and Maintenance

Businesses that are technical in nature require that machinery and equipment should operate at maximum efficiency. Operating at maximum efficiency means that the relevant machinery and equipment should be routinely serviced. There is also the requirement that repairs be carried out on equipment, if breakdown occurs.

In most practical cases, technical support can be arranged for a small business. Technical support can be provided by an engineering firm which can carry out repairs on machinery, generators etc.

SELF ASSESSMENT EXERCISE

List and discus three information about a manufacturing plant that are important to a project evaluator.

Table 10: A Plant/Machinery Checklist

Item	Do you have information on this?
Components of the plant/machine.	
The installed capacity of the machine	
attainable capacity of the machine	
electricity consumption	
Heating power	
Overall weight of machine	
Overall length, width and height.	
Material used e.g. steel	
Structure of the machine	
Screw speed	
Thickness of product to be produced	
Compressed air requirement	
Quality and quantity of	
Water required	
Water temperature	
Electric power voltage	
Electric power capacity	
Type of material feeder	
Take-up force	
Clamping method and force	
Production process	

A Practical Business Idea for a Class Discussion

A Palm Oil Production Plant

Palm oil is an important food item in the Nigerian diet. It is primarily used in cooking and is rich in vitamin A. It is used as industrial raw

material by soap and vegetable oil industries, hence its industrial importance. Put together therefore, we have a product that has a wide market and presents good investment opportunities for a small scale investor.

Technical Considerations

The project considered in this write-up is a medium sized palm oil plant. Since the basic raw material for the project is fresh fruit bunches (FFB) from the oil palm tree, the problem of raw material sourcing is not likely to arise. The basic and necessary machinery for the project are the boiler/steriliser, rotary stripper; digester and clarifier.

The Production Process of Palm Oil

Quartered fruit bunches are introduced into the boiler/steriliser and left for about 30 minutes under hot steam for boiling and sterilisation to take place. The quartered bunches are then fed into the rotary stripper which strips the fruits from the quartered bunches which are still at a very high temperature of about 110°. The loose fruits from the rotary stripper are then fed into the horizontal digester for maceration. In the digester, the palm fruits are beaten into a paste. Hot water which is introduced into the digester forces the thick paste and water into the oil clarifier unit. In the clarifier, the oil is separated from the water and palm oil is collected from a tap. Investors are advised to use the tenera oil palm fruit because of its high oil yielding abilities.

Table 11: A Total Engineering Overview of a Mining Project

	Stage 1	
Rail Link	Roads	Airport
The Mining Complex	Water and Sewerage	Power Generation
	Stage 2	,
Mine	Smelter	Semi-Finished goods
Mine offices	Workshop	Refinery
	Stage 3	
Ore Bin	Ore Crushers	Conveyors
Cranes	Main Ruilding	Sulphuric acid tanks

MBF 739 PROJECT EVALUATION

Table 12: Further Engineering Checklist

Item	Has this been covered in your evaluation?
Site design	
Site engineering	
Surveying	
Quality control	
Drawing control	
Welding supervision	
Safety engineering	
Civil works	
Rigging	
Piping	
Electrical	
Instrumentation	
Mechanical	
Heaters	
Furnaces	
Quantity surveying	
Plan layout	
Plan design	
Site planning	
Security.	

4.0 CONCLUSION

Engineering evaluation is very important in projects that have an engineering content. It helps the project evaluator gain a better insight into the engineering feasibility of a project under evaluation.

5.0 SUMMARY

We have discussed the concept of engineering evaluation. We have seen it as the simple evaluation and break down (in simple terminology) of the engineering feasibility of a project. We discussed the components of engineering evaluation.

6.0 TUTOR-MARKED ASSIGNMENT

Why is the engineering evaluation of a project important?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). A Guide to Small Business Investment. Lagos: Impressed Publishers.

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

MODULE 2

Unit 1	Manpower Planning and Evaluation
Unit 2	Demand Analysis
Unit 3	Supply Analysis
Unit 4	Competition and Marketing Plans
Unit 5	Project Cost Analysis

UNIT 1 MANPOWER PLANNING AND EVALUATION

CONTENTS

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- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Manpower Planning and Evaluation
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 5, we discussed the concept of an engineering evaluation of a project. Engineering evaluation of a project as we saw seeks to evaluate the engineering soundness of a project. This is very crucial especially when the project will be ranked or compared with another project.

Every enterprise requires labour. It is labour that coordinates the other factors of production like land and capital. In terms of project evaluation, our concern is to look at the project and examine the human resources aspects. In terms of manpower planning and evaluation, we need to examine the following:

- Key employees
- The key responsibilities
- The qualifications
- Hours of work
- Training and development of the staff
- Remuneration of the staff

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain manpower planning and evaluation
- discuss the practical applications in industry.

3.0 MAIN CONTENT

3.1 Manpower Planning and Evaluation

In general terms organising manpower in an organisation is the process of assigning duties amongst personnel and coordinating efforts towards the attainment of the firm's objectives. But before organising, there must be a plan. It is the plan that leads to the shaping of an organisation's structure.

Conceptually, the project initiative in structuring the organisation should be concerned about two critical things.

- Job definitions in the project under consideration
- Departmentalisation which follows job definitions.

In doing this, similar jobs are grouped together to form a department. The most common way of organising a project is by function. For example a manufacturing plant may be divided into three functional units namely:

- Production
- Marketing
- Finance

There are two uses of the manpower plan. The first use is by the project initiator. When the project initiator is structuring the project, the manpower plan definitely is a critical component. The project initiator needs to know well in advance who the key employees will be. The key employees will depend on the nature of the business in question. If the business is, for example, soap manufacturing, then a lot of the core production staff should be industrial or pure scientists plus other core support staff. Apart from that, each employee should have their various responsibilities. For example, in a soap plant, you will have core production staff and also quality control staff. They have different responsibilities. And of course, the qualifications of the various staff including their years of experience should be properly documented and evaluated.

The second use of the manpower plan is that financial institutions like banks, before granting loans or overdraft for a project, usually insist on being convinced of the management skills that will be available or are actually available in the firm that seeks to borrow money. They will look at the people concerned, their qualifications and match them with the jobs allocated to them.

Hours of work and the salary and wages of the entire work force are another critical input. The salary and wages of those working on a project is actually expected to hover around the average for the industry.

In practical terms, the manpower of a project can be grouped into two namely:

- Direct labour
- Indirect labour

When we talk about direct labour in a manufacturing project, e.g., a soap plant, we are talking about staff attached to the actual production of the soap. The production manager, production supervisor and factory attendants are all direct labour.

Indirect labour on the other hand are those workers who do not work directly on the manufactured goods but indirectly. They include accounting and admin staff, marketing staff and others.

We have an example of a manpower plan. It contains the following:

- Manpower requirements of the project broken down into direct and indirect labour
- Remuneration of the staff

SELF ASSESSMENT EXERCISE

List and explain four departments that could be found in a big manufacturing firm.

Table 13: Manpower Requirements of the Project

(a) ___Indirect Labour

Post	Specifications	Job Functions
General manager	Degree in sciences,	To oversee the general
	engineering or management	management of the factory
	but with previous experience	
	in a water packaging outfit	
Accounts/admin.	OND Accounting with at	General administration and
officer	least 2 years post	keeping of the books of
	qualification experience	account. Reports to the
		General Manager
Marketing officer	B.Sc or HND Marketing	Reports to the general
	with previous experience in	manager and is in charge of
	marketing of packaged water	marketing of the company
		products
Sales clerk	Senior Secondary School	Reporting to the marketing
	Certificate or GCE	officer, the sales clerk will
		be responsible for all
		clerical duties concerning
		sales
Accounts clerk	Senior Secondary School	Reporting to
	Certificate or GCE	Accounts/admin officer,
		the accounts clerk will be
		responsible for clerical
		accounting duties including
		receipt of cash.
Secretary	OND secretarial studies with	Reporting to the general
	at least 2 years experience in	manager, the secretary will
	a busy organisation	be in charge of all
D .	For Cl. 11	secretarial duties
Drivers	FSLC but with good	Carriage and general
	knowledge of the	distribution of the company
G	environment	products
Security staff	FSLC	General security duties

(b) Direct Labour

Post	Specifications	Job Functions
Production manager	HND water technology	To oversee the
	with at least 2 years	production function in
	experience post NYSC	the water plant
Shift supervisors	OND in science with	To supervise
	relevant experience	production shifts
Technicians	City & Guild/Trade	Maintenance of the
	Test	mechanical and
		electrical components
		of the plant
Laboratory technician	OND Laboratory	Reporting to the
	Science	production manager,
		the laboratory
		technician will be in
		charge of quality
		control
Factory	Senior Secondary	General factory duties
attendants/machine	School Certificate or	including operation of
operators	GCE	the water filing
		machine, pre-market
		packaging of the sachet
		water and loading.

(c) Summary of Manpower Requirements Including Outlays

	No Required	Annual Outlays (N)
Indirect Labour		
General manager	1	420,000
Accounts/admin officer	1	240,000
Marketing officer	1	240,000
Sales clerk	3	360,000
Accounts clerk	2	240,000
Secretary	1	180,000
Driver	3	288,000
Security staff	4	336,000
Sub-Total	16	2,304,000
Direct Labour	1	260,000
Production manager	1 2	360,000
Technicians	2	288,000
Shift Supervisors	4	720,000
Laboratory technician	1	180,000
Factory attendants/machine Operators	12	1,296,000

Sub-Total	20	2,844,000
Grand Total	36	5,148,000
Staff Welfare		514,800
Total Salary,		5,662,800
Wages/Welfare		

4.0 CONCLUSION

This unit has treated manpower planning and evaluation which is a critical aspect of evaluation of projects. The unit has focused attention on the manpower aspects of a project

5.0 SUMMARY

In this unit, we have discussed manpower planning and evaluation. We have seen how manpower planning involves the assigning of duties to personnel and have also tried to relate manpower to a firm's objectives. We also saw that departmentalisation is a critical aspect of a proper manpower plan. Also we discussed the use of the manpower plan. We saw that the manpower plan can be used by two different groups of people – the project initiator and the evaluator.

6.0 TUTOR-MARKED ASSIGNMENT

Why is a manpower plan important in a start-up project?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 2 DEMAND ANALYSIS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 The Demand Analysis Analytical Framework
 - 3.2 Analysis of a Population Data
 - 3.3 Potential Customers
 - 3.4 Evaluating the Size of the Market
 - 3.5 Forecasting Future Demand
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 1, we discussed manpower planning and evaluation. Here we saw the fact that labour is one of the most important aspects of a project whether a start-up or an on-going one. We also used an example to drive home our point.

In this unit, we shall discuss demand analysis. Demand analysis is one first steps towards looking at a market.

Demand analysis is the starting point of our discussions concerning the project and the external environment which is the market. A project is established for a purpose –to serve a market.

The demand analysis is the measurement and forecasting of demand for goods and services of a project. It is like a peep into the future, which as you know, is unknown. A project will survive if the products and services produced by the project are acceptable to the market. Acceptability means that consumers are willing to pay for the goods or services. Demand analysis is a tool used by the project initiator to analyse the total picture of the market that he/she wants to play in.

Likewise, providers of loanable funds like banks also conduct demand analysis to ensure that the projects they wish to sponsor financially have the chances to survive in the market. So from whatever angle you look at it, demand analysis is important.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain what a demand analysis is
- discuss how to conduct demand analysis.

3.0 MAIN CONTENT

3.1 The Demand Analysis – Analytical Framework

To arrange our discussion in an orderly manner, we will try to build an analytical framework that we shall use to do demand analysis. The framework will guide us to take an architectural view of the subject matter. In taking an architectural view, we shall be looking at the larger picture. The analytical framework we shall adopt will follow the sequence below:

- Get total population data and make projections.
- Study the details of the population.
- Define the good or service for which demand analysis is being conducted.
- Define demand again to guide your direction.
- Look at factors affecting demand.
- Using both quantitative and qualitative techniques, generate conditional demand.
- Generate projections for demand using quantitative and qualitative methods.

Table 14: Census 2006: Nigerian Population Data

Abia	2,833,999
Adamawa	3,168,101
Akwa Ibom	3,920,208
Anambra	4,182,032
Bauchi	4,676,465
Bayelsa	1,703,358
Benue	4,219,244
Borno	4,151,193
Cross River	2,888,966
Delta	4,098,391
Ebonyi	2,173,501
Edo	3,218,332
Ekiti	2,384,212
Enugu	3,257,298
FCT	1,405,201

Total	140,003,542
Zamfara	3,259,846
Yobe	2,321,591
Taraba	2,300,736
Sokoto	3,696,999
Rivers	5,185,400
Plateau	3,178,712
Oyo	5,591,589
Osun	3,423,535
Ondo	3,441,024
Ogun	3,728,098
Niger	3,950,249
Nasarawa	1,863,275
Lagos	9,013,534
Kwara	2,371,089
Kogi	3,278,487
Kebbi	3,238,628
Kastina	5,792,578
Kano	9,383,682
Kadunna	6,066,562
Jigawa	4,348,649
Imo	3,934,899
Gombe	2,353,879

3.2 Analysis of a Population Data

We have seen that the starting point of our discussion of a proper demand analysis is to study the population. But we should also understand that getting the total population figure is not the end of the job. We should go further and analyse the population. The purpose of the analysis is to understand certain characteristics of the population. For example, a manufacturer of lipstick has nothing to do with the total population. Rather what is important to the manufacturer is the total female population. There are other aspects of the population that we consider important. Age distribution of a population is one of such aspects. The age distribution of a population is important because it breaks down the population into the various age brackets. Perfect knowledge of the age distribution of a population enables a would-be investor have an idea of his total feasible market well in advance. For example, a manufacturer of baby toys will not be interested in the total population of Nigeria. What will interest the manufacturer is the total number of babies in the population. We have stressed the importance of population data in the understanding of demand analysis. We will go a step further to provide a checklist of further information that is required when we are analysing a population data.

MBF 739 PROJECT EVALUATION

Table 15: Analysis of a Population Data-A Checklist

Item	Have you built this into your demand analysis?
Population of Nigeria by state	
Population by local government area	
Population of Nigeria by sex	
Population distribution by types of	
household	
Population distribution by age	
Population distribution by literacy	
level	
Population distribution by level of	
economic activity	
Population distribution by material	
status	
Population distribution by type of	
disability	
Persons arriving Nigeria by	
nationality	
Persons arriving Nigeria by age group	
Persons departing Nigeria by	
nationality	
Persons departing Nigeria by age	
group	

3.3 Potential Customers

A starting point for our discussion is to properly define who the potential customers of the business are or will be. Customer definition is not an easy task as we shall soon see.

Sometimes confusion arises as to who is a customer because people also fail to distinguish customers from consumers.

The term consumer refers to the person or one who uses a product (or service) for his/her personal satisfaction or benefit. In the industrial sector, the consumer is the one who changes the form of a product to alter its identity. For example, a soap manufacturer will buy caustic soda and use it to manufacture soap. He is the consumer of the raw material. On the other hand, a customer may be a consumer or a dealer who buys a product and eventually resells the product. Also even for a service, the business should clearly identify the types of consumers that they wish to serve. For example, there may be many types of hotels because of the types of customers that need to be served namely:

- hotels for students
- hotels for travellers
- hotels for high-class individuals

Table 16: A Potential Customer Checklist

Item	Has this been taken care of in the plan?
What type of business are you planning?	
What products or services will you offer to	
the market?	
Who is your potential customer?	
How do you define your potential	
customer?	
What are your customer characteristics?	
What are their income levels?	
Who are the consumers of your product or	
service?	
Are the consumers different from the	
customers?	
Where are your customers located?	

3.4 Evaluating the Size of the Market

Apart from identifying who the potential customers of the small business are, there is the need for us to go ahead and evaluate the size of the market for our products or services. The evaluation of the size of the market is very important because it gives us a picture of the total market size in terms of naira and kobo.

A standard analysis should state the market size of the product or service in question. The size of the market is usually estimated by using both qualitative and quantitative methods. To enable us approach our studies carefully, let us follow the following steps in our effort to evaluate market size:

- Define demand
- Examine factors affecting demand
- Examine demand for intermediate and capital goods
- Build market demand.

Defining Demand

For us to be able to properly prepare demand analysis, we need to define demand and understand what we mean by demand.

For purposes of our study, we shall define demand for a product or service as the total quantity of goods or services that able customers in a defined area are willing to buy at set prices. In effect therefore, demand refers to a set of quantities that customers are willing to buy at different MBF 739 PROJECT EVALUATION

prices. In the traditional economic setting, the demand curve is downward sloping (with a price and quantity axis).

It will also be very important for us at this particular time to define and distinguish between two types of demand – total market demand and project demand. Total market demand refers to the total demand within a set geographical area like Nigeria. For example we can make a statement that the total demand for beer in Nigeria is 10 million bottles per annum. The meaning of the statement is that in Nigeria, the total quantity of beer demanded is 10 million bottles at the current ruling prices. But we also need to define another type of demand which we shall call project demand.

Project demand is a subset of the total market demand. If a market for a product or service already exists, the project demand is the project's share of total market demand. If a market is saturated with goods, new projects may face the difficulty of selling their products or services. By and large, it is important to focus attention on a project demand. Mistakes which tend to be fatal occur when analysts confuse market and project demand.

Factors Affecting Demand

We have just defined demand which is very important in our study. But we need to go a step further to examine the factors affecting demand in the market place. It is going to help us build a good demand schedule. In the traditional economic platform, one of the most important factors affecting demand is price. If we assume a well behaved demand curve, then the higher the price of a product, the smaller the quantity that will be bought. Also the lower the price, the greater the quantity that will be purchased. This appears to be a rational behaviour especially where human beings are involved.

Another important factor affecting demand for a product or service is the income of the population. Given a basket of income available to purchasers, it is logical to believe that the higher incomes people receive, the more goods and services they are likely to purchase. So we can say with a measure of confidence that a population with a higher income is more likely to purchase more goods and services than a population with a lower income.

Apart from income, population is another major factor affecting demand in any given setting. It is certain that a large population certainly demands more goods and services than a smaller population. A major aspect of any population which we should not forget is the ethnic and religious structure of the population. The ethnic structure of a population

affects the type of goods and services that will be demanded. For example, in the Nigerian setting, palm oil tends to sell much more in the Southern States than in the North because most Southerners tend to use palm oil in cooking. But in the Northern States, groundnut oil seems to be more available and by implication the people use more of it for their cooking. Also, religion is another aspect of any population that needs to be looked at while drawing up any demand schedule. Again in the Nigerian setting, pork is not an acceptable diet for Muslims but most Christians eat pork. These are the type of critical inputs you need when you are trying to build a demand schedule. Prices of other goods also tend to affect demand. For example, tea and sugar are said to be complementary goods since they go hand in hand. It is therefore logical to say that the price of tea at any given time will affect the demand for sugar. In the same fashion, bread and butter also appear to be complementary goods.

Demand for Intermediate and Capital Goods

When we largely discussed demand our general thinking appeared to be that the goods discussed ought to be consumer goods. But that is not always true. There are some goods which are generally known as intermediate and also capital goods. It is important that we understand these two types of goods.

We shall go ahead to discuss intermediate goods as goods which are not purchased for immediate consumption but rather are used in the production of consumer goods. For example flour is an intermediate good mostly used in the manufacture of bread. Tyres also appear to be intermediate goods if purchased by a car manufacturers. But a tyre is also a consumer product if purchased by Mr. Solomon for use in his car. On the other hand, capital goods are those goods that are used for the manufacture of both intermediate and consumer goods. A manufacturing plant is a capital good. One thing we need to know about capital goods is that they last for very long periods because of their nature.

Building a Market Demand

We have defined what demand for a product or service is generally. Also we understand what a population is.

Let X = No. of persons in a population

Y = average number of a given product consumed by

each person in that population per annum

Then XY = Total market demand per annum for the product in

question

SELF ASSESSMENT EXERCISE

There are 400,000 persons in a city called Madas. Each person in the city drinks a bottle of water daily. What is the annual demand for water in Madas.

3.5 Forecasting Future Demand

Some of us will assume that building the demand schedule for a product is all that is required; but that is not true because the analyst should be able to forecast future demand for the product in question or the service as the case may be. Forecasting future demand appears to be a very difficult task because there is a lot of uncertainty attached to the forecast of a future event. Ordinarily, the demand for many goods may be either stable, erratic or outrightly unpredictable. Social change, structural adjustments, change of taste and technology all add up to make forecasting a difficult task especially of demand.

Usually, there are two basic approaches that are available when trying to forecast demand namely:

- Qualitative techniques
- Quantitative techniques

Qualitative Techniques

Qualitative techniques are mainly desk methods which are based on reports obtained from various sources. One of the sources could be expert opinion obtained from a pool of experts in an industry. The experts usually are seen to be knowledgeable about a product or industry and therefore are in a position to forecast what the future demand for a product is likely to be. Another way of obtaining information for use in forecasting demand is known as the sales force composite method.

With this method, salesmen in a particular industry can provide expert data on demand. But in most cases, the data and the forecast that they provide differ significantly from that provided by a panel of experts. The general belief is that since salesmen operate at the grassroots level, they are more knowledgeable about the demand for products and the accompanying forecast.

Quantitative Techniques

The majority of the data generated are often unreliable, biased and plagued by a subjective state of mind of the people making the forecasts. Also, we think they are clumsy data in the hands of the entrepreneur.

Quantitative techniques in forecasting, however, are less subjective and are more or less based on past data.

In the main they are mathematical in nature and tend to be more accurate. One of the quantitative techniques used is the time series approach. The time series approach generally has become very popular with analysts who are trying to forecast the demand for a good or service. The general feeling is that time is a crucial explanatory variable that explains the future. According to this thinking, past data can explain relationships between the demand of a commodity and time. For example, we can display the relationship between demand and time in a graphical way.

4.0 CONCLUSION

We have fully discussed the demand analysis. We discussed the analytical framework for preparing a demand analysis. We also discussed the analysis of a population data which we said is very crucial. We defined demand and discussed the evaluation of market size and then went further to discuss the forecasting of demand.

5.0 SUMMARY

Understanding demand analysis is our first practical step towards understanding the format of a market.

6.0 TUTOR-MARKED ASSIGNMENT

Why do you think that the analysis of a population data is important when building a demand analysis?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 3 SUPPLY ANALYSIS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 The Supply Equation
 - 3.2 Analysing Supply Data
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 2, we discussed demand analysis which basically looked at the market from the demand side. In this unit, our task is to look at the market from the supply side. Basically when an investor is contemplating an investment in the manufacture of goods for the market, he/she tries to evaluate the nature of the market for the proposed goods or services. Understanding the nature of the market will involve understanding demand as well as supply.

Supply analysis in project analysis tries to focus attention on the supply side of the market. The potential investor will like to know who the current suppliers of the goods or services are. Because this will guide him/her in understanding what is known as the demand/supply gap. Supply analysis tries to identify the supply of given goods or services. It tries to identify who the suppliers are and their locations.

Generally, in measuring the supply of a good, the following should be taken into consideration:

- The domestic supply of the good or service;
- The foreign supply of the good or service. (the imported quantity); and
- The export of the good or service (export quantity).

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain what a supply analysis is
- discuss it in relation to project evaluation.

3.0 MAIN CONTENT

3.1 The Supply Equation

Consider a firm called Abando Nigeria Limited that wishes to set up a liquid soap manufacturing plant in Nigeria. The firm now wants to analyse the supply of liquid soap in Nigeria. As we have said, there are three key items to consider. Those three items are:

- The domestic supply of the liquid soap;
- The foreign supply of the liquid soap;
- The export of the liquid soap in question.

Let A = the domestic supply of a good
B = the foreign supply of a good
C = the export of the good

Then supply X = (A + B) - C.

This is called the supply equation.

You will notice that C is the exported quantity and which will not be available for local consumption.

Table 17: Value of Output of Selected Items

Products	N,000
Food and Drinks	
Flour	4,125,417
Sugar	350,333
Confectionery	1,002738
Biscuits	4,700,211
Beer	3,665,544
Soft drinks	124,668
Wine spirit	84,598
Electrical Equipment	
Refrigerators	244,228
Record Players	n.a
Loud speakers	13,204
Radio cassettes	3,680
Air conditioners	319,361
Television sets	2,528
Electrical cables	48,969
Vehicle Assembly	
Motor cycles	235,633
Passenger cars	1,447,874
Pick up	n.a
Other commercial vehicles	1,078,980

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Building Materials	
Paints	863,479
Cements	2,221,060
Tiles	299,135
Roofing sheets	2,705,982
Steel and iron rods	718,785
Wire-nails and wire	207,520
products	
PVC pipes and fittings	84,149
Petroleum and Chemicals	
Soap detergent	4,618,792
Lubricants	1,673,035
Safety matches	138,841
Batteries	527,551
Textiles	
Cotton textiles	6,679,028
Synthetic fabrics	2,998,946
Knitted fabrics	994,827
Garments	31,477
Paper Conversion	
Exercise Books	260,487
Light and flexible packaging	301,541
corrugated cartons	824,430
Paper bags	384835
Tissue paper	194,928

Table 18: Exports by Products

Description	(N Million)
Live animals and animal products	137.1
Vegetable products	438.8
Food industry products	876.6
Of which tobacco products	0.0
Fats and oil	8.2
Mineral products	4,914,042.1
Chemical and allied products	389.6
Plastic, ethers, esters of cellulose, rubber, etc.	3245.2
Hides, leather and fur	6806.1
Wood, charcoal and wood products	39.6
Paper-making material and articles thereof	621.7
Textiles and textile article	1000.1
Footwear, headgear, umbrellas, feathers, hair	228.8
Stone, plaster, cement asbestos, mica products	42.6
Natural pearls, gemstone and other precious	0.0
metals	
Base metals and articles of base metal	5645.0
Machinery and appliances (other than electrical)	4508.8
Transport equipment	190,409.0

Instruments and apparatus (photos, clocks, etc)	536.4
Miscellaneous manufactured articles	9.1
Arms and ammunitions	0.0
Works of art, collector pieces and antiques	0.1
Other goods and product	40.9

Table 19: Imports by Product Section

Description	(N Million)
Live animals and animal products	108,636.4
Of which live animals	97.7
Vegetable products	103,549.2
Food industry products	84,374.6
Of which tobacco products	14018.3
Fats and oil	4,823.8
Mineral products	88,348.1
Chemical and allied products	133,367.4
Plastic, ethers, esters of cellulose, rubber, etc.	121,732.9
Hides, leather and fur	2,432.4
Wood, charcoal and wood products	4048.3
Paper-making material and articles thereof	44,498.2
Textiles and textile article	27,775.7
Footwear, headgear, umbrellas, feathers, hair	3,258.6
Stone, plaster, cement asbestos, mica products	15,568.2
Natural pearls, gemstone and other precious	44.4
metals	
Base metals and articles of base metal	136,046.2
Machinery and appliances (other than electrical)	409,123.4
Transport equipment	265,034.9
Instruments and apparatus (photos, clocks, etc)	13,464.9
Miscellaneous manufactured articles	9,357.58
Arms and ammunitions	23.3
Works of arts, collector pieces and antiques	6.6
Specials items.	48.4

3.2 Analysing Supply Data

We have agreed that the investor should evaluate the market in terms of determining the level of supply of the goods or service in the market. But in practical terms, there is the urgent need to identify every supplier and the suppliers' location. This will assist in the preparation of a marketing plan. Some analysts believe that getting information on the actual supply data is all that is important. That is not true. In practical terms, there is the need to find out the following facts about supply in an industry:

- No. of players or suppliers in an industry;
- Their current installed capacities;

- Their current actual operating capacities;
- Anticipated expansion plans;
- Critical labour costs in the industry;
- Raw materials costs in the industry and the likely direction; and
- Categorising the suppliers into their various categories big, medium or small players.

SELF ASSESSMENT EXERCISE

Explain the supply equation and its relevance.

4.0 CONCLUSION

The supply analysis attempts to identify the components of the supply of given goods and services as well as the supplies and their locations.

5.0 SUMMARY

The supply analysis seeks to analyse the supply of goods and services. It enables the would-be-investor or analyst to have a clear picture of the supply in the market place. Understanding supply is a must if we are to understand the nature of competition in industry.

6.0 TUTOR-MARKED ASSIGNMENT

Discuss the three components of supply that make up the supply equation.

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 4 COMPETITION AND MARKETING PLANS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Body
 - 3.1 Analytical Framework
 - 3.2 The Nature of Competition
 - 3.3 The Marketing Plan
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

We have discussed market demand when we were looking at market analysis. We also discussed supply. We now want to move ahead and see how both of them interplay in what is known as the market.

In this intellectual journey, we have put ourselves in the shoes of the would-be-investor or analyst who wants to understand the nature of competition in the market. Every firm takes market demand as given and no firm can single-handedly change demand which we said depends on a lot of factors. Also, every firm takes supply to the market as given because no firm is able to control the supply to the market from other competitors.

It is because of this that we really need to understand the nature of competition in the market. Every dynamic environment creates opportunities and problems for firms. A company must be able to respond constructively to this market setting, analyse the opportunities and threats, and then formulate marketing plans to remain in the competition.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of competition
- discuss marketing plans and how they are designed.

3.0 MAIN CONTENT

3.1 Analytical Framework

We know where we are coming from and we also know where we are going. Our task is to discuss the competition and also the marketing plan. To guide us so that we do not forget our direction, we shall propose an outline – a type of framework that should guide us when evaluating competition and also the design of marketing plans.

Marketing plans are very crucial from the point of view of the project initiator as well as from the project evaluator.

Table 20: Competition: A Framework for Analysis

	Demand for the product or service			
	Usefulness and desire			
Industry Outlook	Stability of demand			
	Supply capacity of industry			
	Labour costs			
	Raw material costs			
	Taxation			
	Permits			
	Regulation			
	Market position of the firm in the			
	industry			
Company's Position in the	Products offered			
Industry	Standing of the products			
	The location of the firm			
	Comparative location			
	Relative efficiency of the firm's			
	equipment cost advantages			
	Relative financial strength			
	Ability of company management			

3.2 The Nature of Competition

In every industry, competition exists. It is not a matter of sheer bad luck or coincidence. Competition occurs because every firm in an industry wants to sell its products and also get market share to the detriment of other players.

In the market, the state of competition depends on five basic forces as shown in Figure 5 Therefore, any discussion on competition must take into consideration these five basic forces since they cannot be glossed over. The first force exists within the industry where we see firms all jockeying for position. Here, all firms unleash their strategies and fight

each other. In the end, some firms emerge as clear leaders, some emerge as followers, while others might close shop.

The second force is the threat of new entrants. New entrants into an industry bring in new capacity. But we should note that the threat of entry depends on the barriers present and also the reaction of existing players. We should also not forget the forces arising from suppliers. Strong suppliers can exert strong bargaining power on market participants to the extent of raising prices and influencing the price of goods generally. Powerful buyer groups, when they exist in an industry, tend to influence prices since they more or less dictate the price at which they will buy.

Finally substitute goods or the threat of substitute goods also influence competition in an industry. Substitute goods tend to limit the potentials of an industry. For example sugar tends to limit the honey industry's potentials for growth and expansion.

SELF ASSESSMENT EXERCISE

In evaluating the outlook for an industry, list and discuss four items that you think are important.

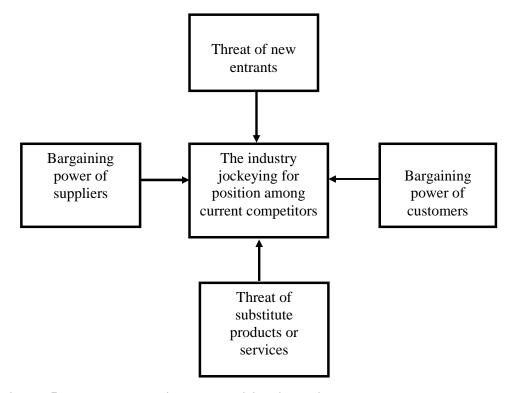


Figure 5: Forces governing competition in an industry

3.3 The Marketing Plan

The marketing plan addresses issues concerning the marketing of the products. It tries to relate the firm to its external consumers and the market.

The marketing plan should answer the following questions:

- What is the product or service?
- What are the uses of the product or service?
- What is the offered price?
- Where will the product be found?
- How will the product or service be advertised?

Table 21: The Marketing Pan Checklist

	Have you covered this in the plan?
How big is the market?	
Who are your competitors?	
How are they faring?	
What is the structure of the	
competition?	
What are your competitors' prices?	
What are your prices?	
What are the trends in the market?	
What market share are you thinking	
of?	

Table 22: The Marketing Plan Checklist

	Have you covered this in the plan?
What are your competitors' strengths?	
What are your competitors'	
weaknesses?	
What is your strength?	
What is your weakness?	
What is your competitive advantage?	
How will you distribute your goods?	
How will you promote your goods?	
What is your customer service policy?	

4.0 CONCLUSION

In this unit, we have discussed competition and marketing plans. We have also discussed the analytical framework for discussing competition. Apart from these, we have provided a check list for the marketing plan.

5.0 SUMMARY

We have discussed competition and marketing plans. We saw how competition and marketing plans relate to each other. The framework for analysis presented discussed industry outlook and a company's position in the industry.

60 TUTOR-MARKED ASSIGNMENT

Discuss the five basic forces that govern competition in industry.

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

Michael E. Porter. (1980). Competitive Strategy: Techniques for Analyzing Industries and Competitors. New York: Free Press.

UNIT 5 PROJECT COST ANALYSIS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Body
 - 3.1 Project Cost Analysis
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 4, we discussed competition and marketing plans. We saw how competition takes place in the market place. We also discussed the components of marketing plans as prepared by project initiators. The marketing plan as we discussed is very important to both the project initiators and the evaluators.

In this unit, we shall discuss project cost analysis which is very important in this course.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain what project cost analysis is
- discuss how the analysis can be prepared.

3.0 MAIN CONTENT

3.1 Project Cost Analysis

Project cost analysis provides total frameworks for calculating or estimating the total cost of a project. For example, a firm wants to set up a garri processing plant to enable it serve the food needs of a growing population. How does the firm know the cost of the envisaged garri processing plant?

To guide our discussions, let us define project cost as all those costs that are incurred in the process of setting up a project. The costs must be attached to the project. And the list of the items must be exhaustive. But we need to arrange the cost items in an orderly and consistent manner so

that like items stay together. To ease our discussions and to make them as easy as possible, we shall divide project cost items into the following sub-headings:

- Cost of land
- Cost of building
- Cost of machinery and equipment
- Cost of utilities
- Cost of furniture and other fittings
- Cost of vehicles
- Pre-operational expenses
- Working capital

Although we have listed the cost sub-headings, we shall go ahead and prepare a small checklist that will guide us. After the checklist, we will work through a practical demonstration using a vegetable oil refining plant as an example.

Table 23: The Cost of Land -A Checklist

	Have you included this in your cost?
Cost of purchase of the land	
Cost of Surveying of the land	
Cost of certificate of occupancy	
Cost of Legal documentation	
Cost of perimeter fence	

Table 24: The Cost of Building -A Checklist

	Have you included this in your cost?
Cost of architectural design	
Cost of structural designs	
Cost of electrical designs	
Cost of factory buildings	
Cost of offices	

Table 25: The Cost of Machinery/Equipment- A Checklist

	Have you included this in your cost?
Cost of locally purchased	
machinery	
Cost of imported machines	
Freight and insurance costs	
Custom duties and other costs.	
Installation and commissioning	
costs	
Test running costs	

Table 26: The Cost of Utilities -A Checklist

	Have you included this in your cost?
Cost of private transformer	
Cost of generator	
Cost of water bore hole with fittings	

Table 27: The Cost of Vehicles -A Checklist

	Have you included this in your cost?
Cost of vehicles for management	
Cost of vehicles for other staff	
Cost of distribution vans	
Cost of distribution Lorries	

Table 28: Working Capital Checklist

	Have you included this in your cost?
Purchase of raw materials	
Purchase of diesel/fuel	
Payment of salaries	
Administration costs	
Selling costs	

SELF ASSESSMENT EXERCISE

List and explain 10 items which you think should appear in a project cost analysis of a start-up sachet water manufacturing plant.

A Worked Example

Analysis of Project Cost in a Vegetable Oil Refining Plant

Estimates of Project Cost Land for the project Civil works and foundations			N 4,000,000 5,000,000
Steel Structures Includes H Beams, U channels, Angles, checker plates, Railing pip Roofing materials	oes,	-	13,000,000
Total Land, Buildings and Steel Structures		-	22,000,000
Storage Tanks			
2 units' crude oil tank – 200 tons		-	3,000,000
1 unit refined oil tank- 300 tons		-	2,000,000
1 unit fatty acid tank – 50 tons		-	850,000
1 unit water storage tank – 20 tons	l.	-	750,000
1 unit diesel storages tank – 20 tor	ıs	-	750,000
1 unit furnace oil tank - 20 tons		-	750,000
Sub-Total	=		8,100,000
Utilities			
1 unit 500 KVA transformer		-	3,500,000
1 unit 500 KVA generator		-	10,000,000
1 unit water bore hole		-	500,000
Sub-Total	=		14,000,000

Machinery and Equipment (Imported)

Full line vegetable oil refining plant consisting of the following:

- Continuous oil pre-treatment section
- Continuous bleaching section
- Continuous physical refining and deodorising section
- Thermal oil heating units
- Water cooling system

Steam generation and distribution system		
		N
Total C & F Lagos U \$ 695,000*N132	=	91,740,000
Bank charges (L/C etc)	=	500,000
Port clearing and other Misc. charge	=	6,500,000
Total: machinery and equipment	=	<u>98,740,000</u>
Pre-Operational Expenses		200,000
Feasibility studies	=	300,000
Project management consulting services	=	500,000
NAFDAC for registration/documentation	=	150,000
Travels and tours (local and overseas)	=	500,000
Total: pre-operational expenses	=	<u>1,450,000</u>
Summary of Project Cost		
Land, building and steel structures	=	22,000,000
Machinery and equipment	=	98,740,000
Storage tanks	=	8,100,000
Utilities	=	14,000,000
Pre-operational expenses	=	1,450,000
Sub-Total	=	144,290,000
Working capital	=	30,617,994
Project Grand Total	=	<u>174,907,994</u>
Analysis of Working Capital Requirements		
		N
One week purchase of raw materials	=	29,697,261
One month factory salary/wages	=	482,820
One month diesel, oil and lubrication expenses	=	437,913

30,617,994

Total

Table 29: A Proposed Financing Plan (N)

_	Project	Bank	Total
	Sponsor		
Land, building and steel	9,000,000	13,000,000	22,000,000
structures			
Machinery and equipment	20,800,000	77,940,000	98,740,000
Storage tanks	-	8,100,000	8,100,000
Utilities	14,000,000	-	14,000,000
Pre-operational expenses	1,450,000	-	1.450,000
Sub-Total	45.250,000	99,040,000	144,290,000
Working capital	30,617,994	-	30,617,994
Total	75,867,994	99,040,000	174,907,994

Contribution Ratio

Project sponsor = 43.3 % Bank = 56.7 % **Total** = **100%**

4.0 CONCLUSION

Project cost analysis is an important aspect of our study of project evaluation. Project cost analysis is important to both the project initiator and the financial analyst who may want to evaluate a project.

5.0 SUMMARY

In this unit, we discussed project cost analysis. In doing this we agreed that cost of land, buildings, machinery and equipment, utilities, furniture and fittings, etc., all form part of the total project cost. We also used a check list to guide the preparation of the cost analysis. Finally we used a worked example of a vegetable oil refining plant to throw more light on the project cost analysis.

6.0 TUTOR-MARKED ASSIGNMENT

Why do you think that it is important to know the total cost of a project?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

Wearne, S.H. (1989). *Control of Engineering Projects*. London: Thomas Telford.

MODULE 3

Unit 1	The Projected Income Statement
Unit 2	Projected Cash Flow Statements
Unit 3	The Projected Balance Sheet
Unit 4	Project Evaluation Criteria
Unit 5	Introduction to Economic Analysis

UNIT 1 PROJECTED INCOME STATEMENT

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Projected Income State
 - 3.2 The Structure in the Projected Income Statement
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 5 of Module 2, we discussed project cost analysis. We discussed how to cost a project, especially a start-up project. We saw that fixed costs and working capital all add up to constitute the project cost. In this unit, we shall discuss the projected income statement which is a very vital statement required for project analysis and evaluation.

The key basis for financial planning and project evaluation is the financial information. The financial information is required to record, compare and evaluate a firm's earning power and ability. In an already existing project, the financial information is already provided since it is a historical data. The income statement or the profit and loss account is a summary of revenues, expenses and net profit of an enterprise for a period of time. This serves as a measure of the firm's profitability over the period. For an on-going project or firm, when prepared, the income statement becomes a historical statement. The projected income statement is a forecast of the revenues, expenses and the net profit of an enterprise or project.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of projected income statement
- discuss its application in project evaluation
- prepare a projected income statement.

3.0 MAIN CONTENT

3.1 The Projected Income Statement

The projected income statement is usually needed by a variety of people. Some of the users of the projected income statement might have direct interest in the firm while others have indirect interest.

The owners or sponsors of a project have a direct interest in the projected income statement. It is so because they are entrusting their investment to the firm. They wish to know before hand what the revenues, expenses and net profit of the firm will be, and most importantly, their own expected dividends.

Another important group that is expected to have a direct interest in the projected income statement of a project is the management. Usually when a project is conceived and a project plan is written, the plan will contain the projected income statement as conceived by the project sponsors or consultants. Usually, the projected income statement is handed down to the project managers as a guide.

Also financial institutions are interested in a projected income statement. Practically, when any firm approaches a financial institution for financial assistance, the firm is expected to prepare a business plan or a project feasibility study which contains, amongst other things, the projected income statement. Financial institutions need to study the projected income statement to evaluate the revenues, expenses and profitability of the investment project. When they do this, they will also test the cash flows of the project to see whether the proposed project can repay any loan granted together with the interest.

Other people that might be interested in the projected income statement are potential investors. Potential investors need to examine the projected income statement to decide whether or not they will invest in a firm.

3.2 The Structure of the Projected Income Statement

We have just explained what the projected income statement is. It is a statement that shows projected revenues, projected expenses, and of course, net profit of a proposed investment, an expansion project or an existing project.

In the standard practice, there is an acceptable arrangement that should group like items together and this leads to building a projected income statement that is broken into revenues, expenses and the net profit.

Revenues

Ordinarily, revenues are the value of output of goods or services that an enterprise supplies to its customers. Revenues, therefore, arise when a firm produces or manufactures goods which it sells to third parties for a fee. Secondly, revenues can arise when a firm is engaged in the buying and selling of goods. It purchases goods which it later resells at a profit or a loss as the case may be. Thirdly, revenues can also arise through provision of services by a firm. A hospital may specialise in surgery and provide surgical services to its customers for whom it collects relevant payments, which when added up, make up the revenues.

Finally, a firm can earn revenue by loaning its economic resources. For example, a bank lends money to customers and earns interest income. The interest earned is revenue.

In projecting for revenues in a project situation, care must be taken so that proper estimates or forecasts are made. And this is made by qualitative judgment plus quantitative judgment on the part of the project evaluator. For example, if the project is a manufacturing facility that will produce goods for the market, the best option is to start with the known market price of the good to be produced. For example, if the good in question is the type of bread that sells for N100 a loaf, then the project evaluator or initiator has to start from the known price of a loaf of bread and that is N100 a loaf. If the number of loaves of bread to be produced per annum amount to 1,000,000 then the projected revenue of the project is N100,000,000.

Likewise, if a firm is engaged in the provision of services, the revenues likely to be earned can be easily estimated. If for example a hospital is projecting revenues, it has to first estimate the likely number of patients that will use its facility and also the average fee it charges a patient. The number of patients multiplied by the average fee per patient will give us the projected revenue of the health facility.

The projection for revenues can cover various periods. In most organisations, revenue projections for project evaluation purposes stretch over a period of three years. Some banks ask for five year revenue forecasts. In the revenue projections care must be taken so as not to overstate the revenues or understate them.

Expenses

The cost of earning revenue is known as the expense. Expenses are different from costs. Cost is the outlay incurred to acquire some asset. For example, when a car is purchased by a company for its business, the sum used to purchase the vehicle is the cost of the vehicle. If the vehicle uses fuel for the firm's operations, that constitutes an expense.

In projecting the expenses of a firm's investment, a lot of factors are usually taken into consideration.

Firstly, we have to get proper estimates of the current cost profile of the various items. For example, when projecting gas and oil expenses of a project, the proper starting point is to collect data on the current prices of gas and oil.

Revenue Projections

From the proposed production plan, the following is the revenue profile for the project in year one (year 2007).

Table 30: Revenue Projection for a Vegetable Oil Refining Plant

Projected Year One Revenues

Product	Quantity	Price Per	Total
	Sold	Ton	Revenue
	(Tons)	N	N
Refined	12,498	145,000	1,812,210,000
Vegetable Oil			
Palm Kernel	18,418	5000	92,090,000
Cake (PKC)			
Palm Kernel	1,315	4000	5,260,000
Sludge (PKS)			
Fatty acid	657.84	100,000	65,784,000
Total			1975,344,000

Summary of Projected Revenues

Revenues (N)

Year 1 – (2007) 1,975,344,000

Year 2 – (2008) 1,975,344,000

Production Materials and Their Expenses

Cracked palm kernel N45, 000 per ton

Bleaching earth N88, 000 per ton

Citric acid N200 per kg

Phosphoric acid N300 per kg

Consumption of Utilities and Chemicals per Ton of Bleached and Refined Vegetable Oil

Steam at 50 psig = 70 kg

Barometric water $= 6 \text{ m}^3$

Clean water in circulation = 7 m^3

Fuel oil = 4 kg

Bleaching earth = 15 kg

Citric acid = 200 gms

Phosphoric acid (for dosing) = 300 gm

Vegetable Oil Packaging Expenses

The refined vegetable oil will be sold in two ways:

1. Direct to vegetable oil distributors who will purchase the vegetable oil in tanker loads. In this case, the vegetable oil tankers will come and load vegetable oil at the factory.

2. The refined vegetable oil will be filled into plastic jerry cans of 9 litre and 18 litre capacity and also sold to the market. The purpose of this is to ensure that the brand of vegetable oil will be in affordable units and prices to the market.

Table 31: Projected Manufacturing Account for a Vegetable Oil Refining Plat

Projected Manufacturing Account for Year Ending 31st December

	2007	2008
Opening raw materials	10,000,000	15,000,000
Raw materials purchased	1,544,257,610	1,544,257,610
	1,554,257,610	1,559,257,610
Raw materials at close	15,000,000	9,000,000
Raw materials consumed	1,539,257,610	1,550,257,610
Add Factory Overheads		
Diesel, oil and lubricant	5,254,959	5,517,707
Factory uniform	110,000	-
Electricity and light	3,721,819	3,907,910
Plant/Machinery repairs	3,002,287	3,152,401
Laboratory consumables	438,820	500,000
Laboratory equipment repair	50,000	80,000
Generator Repairs and maintenance	8,46,556	888,883
Weighbridge fare	290,122	300,000
Salaries and wages	5,793,840	6,083,532
Welding gas	218,499	240,000
Cleaning and sanitation	87,595	90,000
Depreciation	17,248,071	17,248,071
Total factory overheads	37,062,568	38,008,504
Cost of manufactured goods	1,576,320,178	1,588,266,114

Table 32: Projected Expenses for a Vegetable Oil Refining Plant

Projected: Selling and Distribution Expenses

	2007	2008
Selling and Distribution Expenses		
Advertising	5,000,000	5,000,000
Car and bus running expenses	1,782,230	1,871,341
Transports and travelling	2,185,317	2,185,317
Loading and off loading	586,050	586,050
Gifts, entertainment, donations	293,306	293,306
Public relations	418,813	400,000
Total	10,265,716	10,336,014

Administrative Expenses		
Printing and stationery	310,324	325,840
Truck repairs & maintenance	585,862	615,155
Telephone, courier & postages	900,000	900,000
Consultancy fee	120,000	130,000
Security expenses	102,072	107,175
Medical expenses	1,038,632	1,090,563
Audit fee	120,000	120,000
Building maintenance	389,942	409,439
Directors remuneration	7,200,000	7,200,000
Interest and bank charges	16,000,000	12,000,000
Insurance premium	350,000	350,000
Salaries & wages (office)	3,257,100	3,419,955
Depreciation provisions	1,889,544	1,889,544
Total	32,263,476	28,557,671

Table 33: Projected Trading, Profit and Loss Account for a Vegetable Oil Refining Plant

Projected Trading, Profit and Loss Account for the Year Ending 31st December

	2007	2008
Sales	1,975,344,000	1,975,344,000
Opening Stock	30,000,000	40,000,000
+ Cost of Manufactured goods	1,576,320,178	1,588,266,114
Less Stock at Close	40,000,000	50,000,000
=Cost of Sales	1,566,320,178	1,578,266,114
Gross Profit	409,023,822	397,077,886
Deduct		
Selling and distribution expenses	10,265,716	10,336,014
Administrative expenses	32,263,476	28,557,671
Total expenses	42,529,192	38,893,685
Profit before tax	366,494,630	358,184,201
Tax provision	117,278,281	114,618,944
Profit after tax	249,216,3349	243,565,257

• Net Profit

If you deduct expenses from revenues, you will arrive at the profit before tax (PBT).

If you deduct taxation from the profit before tax, you will arrive at the net profit.

SELF ASSESSMENT EXERCISE

List and explain four expense items that can be found in a projected income statement.

4.0 CONCLUSION

We have discussed the projected income statement. We discussed the structure of the projected income statement, revenues, expenses and net profit concepts.

Finally we used as an example to demonstrate a projected income statement.

5.0 SUMMARY

We have treated the projected income statement in this unit. The projected income statement is one of the most important items in project evaluation from the project sponsor's position or from the bank or analyst's position.

6.0 TUTOR-MARKED ASSIGNMENT

Discuss the likely users of a projected income statement.

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 2 PROJECTED CASH FLOW STATEMENTS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Projected Cash Flow Statements Meaning and uses
 - 3.2 The Structure of the Projected Cash Flow Statement
 - 3.2.1 Cash Inflows
 - 3.2.2 Cash Outflows
 - 3.3 Sensitivity Analysis
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 1 we discussed the projected income statement. There we discussed the concept of revenues and expenses and also net profit. We also discussed the fact that the projected income statement is used by a variety of users like the project initiators, bankers and financial analysts.

In this unit, we shall discuss the projected cash flow statements.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain what a projected cash flow statement is
- explain how it can be prepared
- explain the usefulness.

3.0 MAIN CONTENT

3.1 Projected Cash Flow Statement – Meaning and Uses

It is important to understand and analyse the projected cash flows of the firm. We shall begin our discussion by defining a cash flow statement.

A cash flow statement is a statement that shows the actual receipt of cash (inflows) and the disbursement of cash (out flows) of a firm or project. Having said that, we can now go ahead to define a projected cash flow statement.

A projected cash flow statement is a statement which shows the forecasts of actual receipts of cash (inflows) and the disbursement of cash (outflows) of a firm or project. There are many users of information contained in projected cash flow statements. The first user of the projected cash flow statement is the project sponsor or initiator.

The project sponsor or initiator is interested in knowing well in advance the future cash flows of the firm. This is important because the future financing needs of the firm have to be known well in advance. The project initiator needs to distinguish between credit sales and cash sales. If the project initiator does not distinguish between credit sales and cash sales, then his/her project may suffer cash flow problems. The initiator may not be able to estimate the amount of cash needs of the project as well as timing of the cash needs.

Similarly, providers of finance especially the lending banks are usually very interested in the projected cash flow statement. They need to determine the firm's ability to service debt. The debt in question may be existing debts or future debts. Ability to service debts is a function of future cash flows.

Projected cash flow statements assist us to evaluate a firm's future performance and of course financial condition that enables the project evaluator answer the following questions.

- What is the nature of the firm's projected cash flow statement?
- Will the projected cash flow be able to service the project's debts (loan, overdraft + interest)?
- When will the project need financing and to what extent?
- How should the loan or overdraft or finance be structured?
- How stable are the cash flows?

3.2 The Structure of the Projected Cash Flow Statement

The basic format of the projected cash flow statement is displayed in Table 33 is a projected cash flow statement of a company. But it covers a period of only 3 months. You can project a cash flow as long as you require but the basic principles should be followed. If you examine Table 33 properly, you will realise that the projected cash flow statement is divided into two main sections, namely:

- 1. The cash inflows
- 2. The cash outflows (outgoings)

We will now go ahead to break down the projected cash flow statement.

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Table 34: A Three Month Projected Flow Statement Niger Limited

Cash Inflows	January	February	March
Capital introduced	10,000,000	-	1
Loan introduced	20,000,000	-	1
Cash sales	40,000,000	50,000,000	60,000,000
Total Cash Inflows	70,000,000	50,000,000	60,000,000
Cash Outflows			
Raw material	30,000,000	30,000,000	35,000,000
Salary and wages	2,000,000	2,200,000	2,500,000
Office admin expenses	500,000	550,000	600,000
Electricity and gas expenses	500,000	550,000	600,000
Loan repayment	2,000,000	2,000,000	2,000,000
Interest charges	400,000	400,000	400,000
Selling expenses	1,000,000	1200,000	1,300,000
Total Cash Outflows	36,400,000	36,900,000	42,400,000
Cash	33,600,000	13,100,000	17,600,000
Surplus/(Deficit)			
Opening Cash Balance	-	33,600,000	46,700,000
Closing Cash Balance	33,600,000	46,700,000	64,300,000

3.2.1 Cash Inflows

We have seen that a projected cash flow statement is broken down into the inflows and the outflows (outgoings). Let us now proceed to examine some of the key items contained in the projected cash flow statement. The items will vary from capital to loan introduced and also cash sales. We shall treat them individually.

• Capital Introduced

Every firm or project should have a capital. At the time a project is conceived or is being expanded, the owners of the firm usually bring in what is known as capital.

In a limited liability company, the share holders usually contribute the capital of the firm. In cash flow construction, capital is usually entered as an inflow. The reason is clear. When you introduce capital, you bring in cash.

• Loan

Another item appearing in a projected cash flow statement is loan. In some cases, a project is funded through loan from banks.

The loan will usually have the following features:

- 1. The loan amount will be specific
- 2. The loan has an interest rate attached to it.
- 3. The loan will be repaid in agreed installments.

Cash Sales

The sales figure is the most important in a projected cash flow statement. Projections for sales pose one of the most difficult challenges in cash flow projections. We must quickly distinguish between total sales, credit sales and cash sales. Total sales are the total value of goods or service sold to third parties. Credit sales refer to sales for which payment is not made immediately. The figure for credit sales is usually transferred to the debtors list. Cash sales are the difference between total sales revenue and credit sales.

As far as we are concerned, the cash sale is the most important component of sales and it is the one that appears in the projected cash flow statement. Credit sales are only reflected in the cash flow when they are converted to cash. For example, if in January 2007, a company sells four cars at a credit of N10,000,000. In the cash flow for January 2007, there will be no entry for cash sales. But if in February 2007, the company receives a cash payment of N5,000,000, then that figure will appear in the inflows column for February 2007.

The basic rule is that only actual cash received is usually entered in the inflow column.

In actual practice, projecting for cash sales will involve exhaustive consideration of the following:

- General economic outlook in the country
- The industry outlook. What is the demand situation like? What is the supply situation?
- What is the structure of competition and how fierce is it?
- What will be the effect of competition on prices in the firm's area of operation?

3.2.2 Cash Outflows

Cash outflows or outgoings will include all expenses that use cash. They will include items like:

- Raw material expenses
- Salary and wages
- Stationery
- Loan repayment
- Interest charges
- Selling expenses
- Office admin expenses
- Oil and gas expenses
- Taxation
- Rates and permits

3.3 Sensitivity Analysis

Usually, when constructing a projected cash flow statement, the first set of projections is what we call normal estimates of cash flows. Normal estimates of cash flows especially cash revenues are based on all things being equal; but all things cannot be equal. A lot of things may happen.

In a market, supply can come from unexpected source and cause prices to fall. Prices of raw materials may rise suddenly and all these tend to reduce our earlier revenue projections and jack up expenses.

Sensitivity analysis provides the tool for subjecting a project's cash flows to adverse market situations. Sensitivity analysis seeks to adjust revenues for risk and also costs. In conducting sensitivity analysis, we say that we are adjusting a project's cash flows for risk. If we conduct sensitivity analysis on a cash flow, we may do that by making one, two or three of the following assumptions:

- Due to intense competition the project will not be able to make the earlier normal sales volume. Cash revenues will drop.
- Due to excess supply, prices in the market will fall, that will reduce cash revenues.
- The prices of raw materials and other items will rise. A close examination will reveal that the impact of any of the above will have the effect of reducing the cash revenues of a project.

We now state that if a normal projected cash flow statement is reconstructed to accommodate the fact that the market could be worse, we say that the reconstructed cash flow is now called a risk "adjusted cash flow statement." The risk- adjusted cash flow is a pessimistic cash flow and should be admired by analysts.

Table 34 is a risk-adjusted income statement reconstructed from Table 33.The critical assumption is that Table 34 assumes that only 75% of cash sales of Niger limited will be realised.

Sensitivity analysis is a tool for subjecting cash flows to risk analysis. The key objective of the sensitivity analysis is to forecast a worst-case scenario for a project.

Other ways of conducting a sensitivity analysis is to assume that expenses attached to a project will increase.

Table 35: A Three Month Risk-Adjusted Projected Cash Flow Statement Niger Limited

Cash inflows	January	February	March
Capital introduced	10,000,000	-	-
Loan introduced	20,000,000	-	-
Cash sales	30,000,000	37,500,000	45,000,000
Total Cash Inflows	60,000,000	37,500,000	45,000,000
Cash Outflows			
Raw material	30,000,000	30,000,000	35,000,000
Salary and wages	2,000,000	2,200,000	2,500,000
Office admin	500,000	550,000	600,000
expenses			
Electricity and gas	500,000	550,000	600,000
expenses			
Loan repayment	2,000,000	2,000,000	2,000,000
Interest charges	400,000	400,000	400,000
Selling expenses	1,000,000	1,200,000	1,300,000
Total Cash	36,400,000	36,900,000	42,400,000
Outflows			
Cash			
Surplus/(Deficit)	23,600,000	600,000	2,600,000
Opening Cash	-	23,600,000	24,200,000
Balance			
Closing Cash	23,600,000	24,200,000	26,800,000
Balance			

4.0 CONCLUSION

We have discussed projected cash flow statements. We discussed the nature of cash flow statements and their users. We also examined the structure of cash flow statements. We used an example to show what a projected cash flow statement looks like. We also constructed a risk-adjusted cash flow statement.

5.0 SUMMARY

In this unit we treated projected cash flow statements which we said constitutes a very vital document used in the evaluation of projects. The cash flow gives us a picture of cash inflows and outflows together with timing.

6.0 TUTOR-MARKED ASSIGNMENT

Why do you think that banks are interested in projected cash flow statement of projects?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 3 THE PROJECTED BALANCE SHEET

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Meaning of a Balance Sheet
 - 3.2 Components of a Balance Sheet
 - 3.2.1 Assets
 - 3.2.2 Liabilities
 - 3.3 Construction of the Projected Balance Sheet
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

In Unit 2, we discussed the projected cash flow statement. There, we discussed inflows and outflows (outgoings) of a firm or project. We also discussed the structure of the projected cash flow statement and went a step further to provide an example of a projected cash flow statement.

In this unit, we shall discuss the projected balance sheet.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of a projected balance sheet
- explain the preparation of the projected balance sheet
- prepare a projected balance sheet.

3.0 MAIN CONTENT

3.1 Meaning of a Balance Sheet

Before we delve into the projected balance sheet proper, it is very important for us to first understand what a balance sheet is.

The balance sheet or the statement of financial position is one of the most important financial statements. It shows the financial condition or better still, the statement of affairs of a firm or business. We will

therefore, define a projected balance sheet as a forecast of a future balance sheet as at a future date.

3.2 Components of the Balance Sheet

The balance sheet has two main sides namely:

- Assets
- Liabilities

3.2.1 Assets

When we are talking of assets generally, we are talking about the valuable possessions owned by the firm, valued in monetary terms. They will include land and buildings, stock of goods, raw materials, cash, vehicles and other valuables.

But generally we can classify assets under the following headings:

- Current assets
- Investments
- Fixed assets

Lets us now discuss each of them:

Current Assets

The current assets of a firm or business are those assets which are held in the form of cash or expected to be converted into cash in a short period or within the accounting period of the firm. In actual practice, the accounting period is usually of a one-year duration.

The current assets of the firm will include the following:

- Cash
- Book debts (debtors).
- Prepaid expenses
- Marketable securities.
- Stock

Let us start with cash which is one of the most liquid current assets. Cash will mean cash on hand or cash in the bank.

Another current asset which is important is book debts (debtors). Book debts are sometimes called account receivables. These are amounts due from debtors to whom goods have been sold or service rendered. Some of the book debts may be realised by the firm. If they are not realised they turn into what is called bad debts and may be written off later.

Prepaid expenses are also current assets. They are expenses of future periods that are paid in advance. An example of prepaid expenses is rent which may be payable in advance by a firm. For example in January 2007, a firm may pay rent for its office for January 2007 to December, 2007. If in April, 2007, the financial year of the firm ends, it will regard the portion of rent paid from May 2007 to December, 2007 as a prepaid expense which invariably is a current asset.

Stock (inventory) is another current asset and includes raw material, work in process and finished goods. The raw materials and work in process are required for maintenance of the production function of the firm.

Finished goods usually will be already packed and kept ready for purchase by customers of the business.

Marketable securities are the firm's short term investment in shares, bonds and other securities. The securities are usually marketable and can be converted into cash in a very short time.

Investments

Investments represent the firm's investments in shares, debentures and bonds of either firms or the government. By their nature, the investments are long term.

It is important to note that the investments yield income to the firm.

Fixed Assets

Fixed assets are long-term assets held for periods longer than one year. They are usually held for use in the firm's business. Fixed assets include land, buildings, machinery and equipment, vehicles, etc.

We have briefly seen what the assets are. We shall now move over and discuss liabilities.

3.2.2 Liabilities

When we talk of liabilities, we mean the debts that are payable by the firm or business to creditors. They may represent various obligations due to various third parties arising from various business transactions. Examples of liabilities include creditors, accounts payable, taxes payable, bonds, debentures, etc.

But generally, liabilities are divided into two broad groups namely:

- Current liabilities and
- Long-term liabilities

We shall discuss each of the groups

Current Liabilities

Current Liabilities are those debts that are payable in a short period usually within a year.

One of the major current liabilities is the bank overdraft. Most banks grant their customers overdraft which are repayable within a period of one year. The other type of current liability includes provisions for taxes and dividends. These are liabilities that will mature within one year. Another type of liability is expenses payable. The firm may owe expenses to public power supply organisation or have rents to be paid.

Long Term Liabilities

Long-term liabilities are the obligations which are payable in a period of time greater than a year.

One of the long term liabilities of a firm is term loan. The firm may borrow money from a bank that will be repayable over a period exceeding one year. Such a borrowing or loan is regarded as long-term liability.

Also, when a firm needs to raise a large sum of money, it issues debentures. A debenture is an obligation on the part of a firm to pay interest and principal under the terms of the debenture.

However one of the most stable types of long term liability is owners' equity. Owner's equity represents the owners' interest in the firm. In practical terms, the total assets of a firm less the liabilities are the owners' interest. The owners interest in the firm consist of

- Paid up share capital and
- Retained earnings (undistributed profits).

SELF ASSESSMENT EXERCISE

Discuss the components of a balance sheet.

3.3 Construction of the Projected Balance Sheet

In the earlier sections of this unit, we have discussed the balance sheet generally. That was from a historical perspective. We shall now discuss the construction of a projected balance sheet.

The following steps are recommended:

- Start from the determination of sales revenue.
- Compute cost of goods sold (COGS)
- Compute admin expenses, general and selling expenses.
- Bring forward sundry income and expenses and generate the projected income statement.
- Determine taxation, dividends and retained earnings.
- Project for assets.
- Project for liabilities.

Table 36: A Projected Balance Sheet

Projected Balance Sheet (N)

As at	Year 1
Assets Employed	
Fixed assets	66,629,024
Preliminary expenses	33,140
Total	66,662,164
Current Assets	
Stock-in-trade	12,000,000
Raw materials	12,000,000
Debtors and prepayment	1,000,000
Cash and bank balance	2,623,497
Total Current Assets	27,623,497
Current Liabilities	
Creditors and accruals	2,000,000
Tax provisions	10,247,185
Total current liabilities	12,247,185
NET CURRENT ASSETS	15,376,312
Total Assets	82,038,476

Financed by	
Share capital	
Loan	
Profit and loss account	
Directors current account	
Total Liabilities	

Table 37: Projecting for Assets in a Balance Sheet Checklist

Item	Have you covered this in your projection?
List existing plants and depreciation	
Determine additions to Plants and machinery.	
Determine new depreciation Determine capital allowances Determine changes in the Investment account	
Bring forward all sundry assets according to previous years data	
Estimate cash Estimate marketable securities	
Total the Asset side of The balance sheet.	

Table 38: Projecting for Known Liabilities in a Balance Sheet Checklist

Item	Have you covered this in your projection?
Record account payable and determine the trend.	
Record accruals and project for increases or decreases according to historical patterns	
Record long term debt and project for the future balances bearing in mind the repayments.	

Record leases if any and project for the future amortisations.

Bring forward other sundry liabilities,

Estimate taxation based on the projected income statement and forecast the future trend.

From the net profit estimate amount going to dividends and retained earnings.

Record retained earnings. Record paid up capital. Fine tune grey areas.

Total the liabilities to agree with total assets.

4.0 CONCLUSION

We have discussed the projected balance sheet. We first discussed assets generally and then went ahead to discuss liabilities. We discussed the construction of the projected balance sheet and provided a checklist for the projection for both assets and liabilities.

5.0 SUMMARY

We have treated the projected balance sheet. The projected balance sheet as we discussed is a forecast of a future balance sheet as at a future date. It will show what the assets will be and also what the liabilities will be. It is a very important document in project evaluation.

6.0 TUTOR-MARKED ASSIGNMENT

- 1. Who do you think are the users of the information in a projected balance sheet?
- 2. Why do they need the information contained in it?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). Project *Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 4 PROJECT EVALUATION CRITERIA

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Traditional Criteria of Project Evaluation
 - 3.2 The Discounted Cash Flow (DCF) Method
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

1.0 INTRODUCTION

Let us recall that the focus of this course is project evaluation. From unit 1, we discussed the project cycle. From there we moved on to discuss factors affecting location of projects. We also discussed capacity and production planning, demand analysis, supply analysis, project cost analysis, projected income statements, cash flows and the balance sheet.

All these have set the stage for us to tie the discussions. We now want to discuss a very crucial aspect of this course, which is the project evaluation criterion. Project evaluation criteria seek to present the methods to be adopted to measure the value of an investment project. The evaluation enables us to choose between two or more projects once the values are known.

Any project evaluation criterion to be adopted should posses the following characteristics:

- It should provide a means to distinguish between acceptable and unacceptable projects.
- It should also be able to rank projects in order of their desirability.
- It should be a criterion that is applicable to any conceivable project.
- It should recognise that bigger cash flows are preferable to smaller ones.
- It should recognise that early cash flows or benefits are preferable to later cash flows or benefits.

Although there are a lot of project evaluation criteria in the literature, we shall discuss the most widely accepted criteria which are the traditional criteria and the discounted cash flow (DCF) criteria.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- discuss project evaluation criteria
- distinguish between the traditional criteria and the discounted cash flow relative to project evaluation.

3.0 MAIN CONTENT

3.1 Traditional Criteria of Project Evaluation

In the traditional criteria, we shall discuss two methods, namely: the payback period and the accounting rate of return method.

The Payback Period

The payback period is one of the most popular methods of project evaluation. The payback period is defined as the number of years required to recover the original cash outlay invested in a project. If the project yields constant annual cash inflows, the payback period can be computed by dividing cash outlay by the annual cash inflow.

So we say thus:

Payback period = Cash outlay (investment)

Annual Cash inflow

Example

A project requires a cash outlay of N200,000 and yields an annual cash inflow of N50,000 for a period of 10 years; calculate the payback period.

The payback period is $\frac{N200,000}{N50,000} = 4$ years.

However, it is to be noted that in the case of unequal cash inflows, the payback period can be computed by adding up the cash inflows until the total is equal to the initial cash outlay.

The payback period is greatly admired by project evaluators because it is very simple to understand. Another good virtue of the payback period is that it costs less than most of the other sophisticated methods.

However, despite its simplicity, the payback period may not be a desirable investment criterion. In the first place, it fails to recognise the cash flows that come in after the payback period. Again it fails to consider the pattern of cash inflows and that early cash inflows are better than later cash inflows.

Despite its weakness, the payback period is very popular amongst analysts. It tries to emphasise early recovery of an investment. This means that it gives an insight into the cash inflows of the project.

The Accounting Rate of Return (ARR) Method

The accounting rate of return (ARR) is a method that uses accounting information to measure the profitability of an investment.

The accounting rate of return (ARR) is computed by dividing the average income after taxes by the average investment.

ARR <u>Average Income</u> = Average Investment

Example

A project costs N100,000 and has a scrap value of N40,000. The stream of income before depreciation and taxes are N40,000, N50,000 and N60,000 for the first three years. The tax rate is 50% and depreciation is on straight line basis.

Calculate the accounting rate of return for the project.

Solution

	Year	Year	Year
	1	2	3
	N	N	N
Earnings before depreciation	40,000	50,000	60,000
and taxes			
Depreciation	20,000	20,000	20,000
Net earnings before taxes	20,000	30,000	40,000
Taxes at 50%	10,000	15,000	20,000
Net earnings after taxes	10,000	15,000	20,000

Book value of investment

Beginning	100,000	80,000	60,000
Ending	80,000	60,000	40,000
Average	90,000	70,000	50,000

Average earnings =
$$\frac{10,000+15,000+20,000}{3}$$

= 15000

Average investment =
$$90,000 + 70000 + 50,000$$

= 70000

Accounting rate of return =
$$\frac{15000}{70000}$$
$$= 21.42\%$$

As an accept or reject criterion, the ARR method will accept all those projects whose ARR is greater than the minimum rate established by management. If the ARR is lower than the minimum rate established by management, then the project should be rejected.

The ARR method is very simple to understand and use. It can also be easily calculated using accounting information.

However, the ARR suffers from three main weaknesses. First it uses accounting profits not cash flows in appraising projects.

Secondly ARR ignores the time value of money. The profits occurring in different periods are valued equally.

Thirdly, it does not allow the fact that profit can be reinvested to earn more profits.

3.2 Discounted Cash Flow (DCF) Methods

We have discussed two of the traditional methods used in the evaluation of projects. One is the payback period while the other is the accounting rate of return (**ARR**). Although two of them are simple to use and understand, they are not theoretically sound. Both of them fail to consider the timing of cash flows. Both fail to consider the time value of money.

Because of these limitations, we shall consider two superior investment criteria which fully recognise the timing of cash flows.

The two methods are the net present value (**NPV**) method and the internal rate of return (**IRR**) method. These two methods are referred to as discounted cash flow (**DCF**) methods or the tme-adjusted methods.

The Net Present Value (NPV) Method

This method correctly recognises the fact that cash flows arising at different time periods differ in value and are comparable only when their equivalent- present values are found out.

The following steps are followed when computing the net present value (NPV).

- 1. A discount rate is selected to discount the cash flows. The correct discount rate should be the firm's cost of capital which is the minimum rate of return expected by the investors to be earned by the firm.
- 2. The present value of cash inflows and outflows are computed using cost of capital as the discounting rate.
- 3. The net present value (NPV) is the present value of cash inflows less present value of cash outflows.

The acceptance rule using the NPV method is to accept a project if the NPV is positive, and to reject it if the NPV is negative.

If NPV is greater than zero, then the value of the firm is expected to increase. It is also important for us to understand the interpretation of NPV. The net present value may be interpreted to mean the immediate increase in the wealth of a firm if the investment proposal is accepted. It is equal to an unrealised capital gain. The net present value can also be interpreted to represent the amount the firm could raise at a required rate of return in addition to the initial cash outlay to distribute immediately to its shareholders and by the end of the project life to have paid off all the capital raised plus interest on it.

Example

Calculate the net present value of a project which cost N500,000. but generates cash inflows of N150,000, N300,000 and N400,000 over a three year period. The required rate of return is 10%.

Solution

Year	Cash inflows	Discount factor	Present Value
		at 10%	of Cash Inflows
	N		N
1	150,000	.909	136,350
2	300,000	.826	247,800
3	400,000	.751	300,400

Total	684,550
Less investment outlay	500,000
Net present value	184,550

In terms of merit, the NPV method is very significant since it recognises the time value of money. It also is consistent with the objective of maximising the wealth of shareholders. However, the NPV suffers from the following limitations.

Firstly, it is fairly difficult to use.

Secondly, in computing the NPV, it is assumed that the discount rate which usually is a firm's cost of capital is known. But as we know, the cost of capital is a fairly difficult concept to measure in real life.

Thirdly, NPV may not yield a consistent answer when the projects being compared involve different amounts of investment.

The Internal Rate of Return (IRR) Method

The internal rate of return (IRR) can be defined as that rate which equates the present value of cash inflows with the present value of cash outflows of an investment. Put in another way, the internal rate of return is the rate at which the NPV of an investment is zero. It is called the internal rate because it depends solely on the outlay and the resulting cash inflows of the project and not any rate determined outside the investment.

Let C	=	Cash outlays of an investment
<u>A1</u> (I+R).	=	Cash in flows received in year I discounted at the cost of capital R.
A2 (I+R)2	=	cash inflows received in year 2 discounted at the cost of Capital R.
<u>A3</u>	=	cash inflows received in year 3

Write the basic equation

C =
$$\frac{A1}{(1+R)}$$
 $\frac{A2}{+(1+R)^2}$ + $\frac{A3}{(1+R)^3}$

O = C -
$$\frac{A_1}{(1+R)}$$
 + $\frac{A_2}{(1+R)^2}$ + $\frac{A_3}{(1+R)^3}$

The value of R in the equation at which total cash outlays equal total cash inflows is called the internal rate of return (IRR).

Usually the value of R can be found out by trial and error. Generally, if the calculated present value of the expected cash inflows is lower than the present value of cash outflows, a lower rate should be tried. On the other hand, if the calculated present value of the expected cash inflows is higher than the present value of cash outflows, a higher rate should be tried.

Example

A barbers' shop costs N32,400 to establish and is expected to generate cash inflows of N16,000,N14,000 and N12,000 over its life of three years. Calculate the internal rate of return.

Solution

Let us start by trying 16%

Year	Cash Inflow	Discount	Present Value
		Factor at 16%	
	N		N
1	16,000	.862	13,792
2	14,000	.743	10,402
3	12,000	.641	7,692

Total 31,886 Less cash outlay = 32,400 Net present value (NPV) = -514

The net present value is -N514 at 16% discount factor. Let us try a lower rate like 14%

Year	Cash Inflow	Discount	Present Value
		Factor at 14%	
	N		N
1	16,000	.877	14,032
2	14,000	.769	10,766
3	12,000	.675	8,100

Total 32,898
Less cash outlay = 32,400
Net present value (NPV) = 498

You will observe from the above calculations that when we tried 16% discount rate, the NPV was negative at -N514, when we tried 14% discount rate, the NPV became positive at N498. Therefore, the internal rate of return we are looking for lies between 14% and 16%.

The basic accept-or-reject rule, using the IRR method, is to accept the project if its internal rate of return is higher than the firm's required rate of return. However, the project should be rejected if its internal rate of return is lower than the firms cost of capital.

It is important that we understand the interpretation of the internal rate of return (IRR).

The internal rate of return (IRR) represents the highest rate of interest a firm would be ready to pay on funds borrowed to finance the project without being financially worse-off, by repaying the loan principal plus accrued interest out of the cash inflows generated by the project.

We should also see the internal rate of return method as a very sound method. As we said, it is a discounted cash flow method and also it considers the time value of money. It is also compatible with the firm's desire to maximise the owners' wealth. However the IRR method is fairly difficult to understand and it involves complex computations.

SELF ASSESSMENT EXERCISE

Distinguish between the traditional project evaluation methods and the discounted cash. Show criteria.

4.0 CONCLUSION

We have discussed project evaluation criteria which we said constitute a very crucial topic in this course. We discussed traditional criteria of project evaluation. Here we mentioned the payback period and the accounting rate of return (ARR). We also discussed discounted cash flow criteria. Here we mentioned the net present value (NPV) method and the Internal Rate of Return (IRR).

5.0 SUMMARY

Project evaluation criteria provide us with the tools with which we can choose from various investment proposals using acceptable techniques. The evaluation criteria guide the project initiator and assist him/her to choose among alternative projects. Also banks use project evaluation criteria to decide whether or not to lend money for a project.

6.0 TUTOR-MARKED ASSIGNMENT

Why are the discounted cash flow (DCF) techniques better and more acceptable than the traditional methods of project evaluation?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.

UNIT 5 INTRODUCTION TO ECONOMIC ANALYSIS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Financial Analysis and Economic Analysis- A Comparison
 - 3.2 The Nature of Economic Analysis
 - 3.3 Adjustments to Financial Analysis
 - 3.4 Linkage Effects of a Project
- 4.0 Conclusion
- 5.0 Summary
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1.0 INTRODUCTION

Generally, in a project analysis situation, most analyses focus on the cash inflows and outflows of a project. Critical expenses and incomes are usually compared to determine whether a project should be undertaken or not. But expenses and revenues in most financial analyses are mainly the consideration of a private investor.

The implication of financial analysis is that it provides a micro view of a project and concentrates attention on things like accounting profits.

Economic analysis on the other hand considers projects from a macro point of view. The type of questions asked in an economic analysis are:

- 1. Will the project under consideration lead to the general well being of the community, the state and the nation?
- 2. Will the project generate employment at various levels in the macro environment?
- 3. Will the project lead to economic growth?
- 4. What are the linkages that the project has, i.e., forward or backward linkages?
- 5. Will the project generate more technical knowledge?

The questions that we have asked are not exhaustive but only go to demonstrate the type of questions that economic analyses seek to answer.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of an economic analysis
- distinguish between an economic analysis and a financial analysis.

3.0 MAIN CONTENT

3.1 Financial Analysis and Economic Analysis – a Comparison

In general theory, a financial analysis tries to solve resource allocation problems. It tries to use information from projects to determine whether projects should come on stream or not.

Economic analysis also tries to solve resource allocation problems in an economy. In economic theory, resources are very scarce and it is part of any good analysis to allocate resources between competing projects. For example, resource allocation problems can arise if a community is trying to decide whether to build a school or a hospital with limited scare resources.

Financial analysis equally tries to allocate resources but from a micro view point. So, both financial and economic analyses solve resource allocation problems.

Financial analysis tries to concern itself with issues of both benefits and costs arising from a project. In the financial analysis, the concern of the analysis is to evaluate the stream of costs attached to a project and deduct same from the stream of benefits.

If the stream of benefits is greater than the stream of costs, then the project in question has a positive value and should be accepted, all things being equal. However, if the stream of costs is greater than the stream of benefits, then the project in question has a negative value and should not be accepted, all things being equal.

Economic analysis also concerns itself with costs and benefits arising from a project. If the stream of benefits is greater than the stream of

costs, then the project in question has a positive value and should be accepted.

However, if the stream of costs is greater than the stream of benefits, then the project in question has a negative value and should not be accepted, all things being equal.

So we could say that financial analysis and economic analysis both concern themselves with costs and benefits arising from a project. In the end, they provide answers to the question of whether a project should be acceptable or not. In evaluating projects, both use discounting and compounding techniques to arrive at their answers.

However, there exist conceptual differences between financial analysis and economic analysis. While financial analysis has a primary objective of establishing the viability and acceptability of a project from a financial view point, paying no attention to society, economic analysis has the objective of establishing the fact that a project is acceptable or not to the society as a whole. So while financial analysis has a micro objective, economic analysis has a macro objective.

Finally, in reaching a decision as to whether or not to accept a project, financial analysis and economic analysis both try to establish a relationship between costs and benefits.

For example in financial analysis, costs and benefits arising from a project are usually defined in monetary variables such as profits. But economic analysis goes really beyond the vague definitions of profit. In Economic analysis, costs are defined in terms of opportunity costs or foregone costs to the society as a whole.

SELF ASSESSMENT EXERCISE 1

Compare and contrast financial analysis and economic analysis.

3.2 The Nature of Economic Analysis

In economic analysis, the costs and benefits attached to a project are usually compared before a decision can be reached on whether or not to accept a project.

In the literature, there exist three discounted measures of project worth which we will now discuss:

The Net Present Worth

The net present worth is the difference between the present worth of benefits and the present worth of costs. We can write thus:

Net Present Worth = Present Worth of benefits Present Worth of costs

Generally, according to the net present worth theory, a project is acceptable if the net present worth is positive. If the net present worth is negative, the project will be rejected.

Benefit-Cost Ratio

If you divide the present worth of benefits of a project by the present worth of its costs, then you have what is known as the benefit-cost ratio.

We can write thus:

Benefit-Cost ratio = Present worth of benefits
Present worth of costs

Generally, a project is acceptable if the benefit-cost ratio is greater than 1 (one).

If the benefit-cost ratio is exactly 1 (one), that project is a marginal project or break even project.

The Internal Rate of Return (IRR)

The internal rate of return is a discount rate where the present worth of benefits is equal to the present worth of costs.

Under the internal rate of return evaluation method, a project will be acceptable if its internal rate of return is higher than the firm's required rate of return.

The starting point of economic analysis is the financial analysis of projects which should be properly concluded before embarking on an economic analysis. Some adjustments will be made to the financial calculations to arrive at economic data.

First, it may be necessary to include or exclude some costs and benefits which may have been included or excluded from the financial analysis.

Secondly, some project inputs and outputs may have to be revalued if their shadow prices differ significantly from their market prices.

SELF ASSESSMENT EXERCISE 2

Discuss the nature of economic analysis with emphasis on the methods of evaluating the worth of a project

3.3 Adjustments to Financial Analyses

We have stated that the starting point of an economic analysis is a financial analysis, so if we have financial data on financial analysis, we need to make some adjustments to the financial analysis to arrive at economic analysis data. We shall now consider some of the adjustments:

Transfer Payments

Transfer payments represent transfer of resources from one section of society to another. They do not make any claim on the country's resources and as such, their impact should be clearly distinguished and analysed in the economic analysis.

One of the first transfer payments we shall consider is interest. Interest is a reward for capital. For example, if a project is funded through a bank loan, the interest component is included in the profit and loss statement. The interest charges in the profit and loss statement represent transfer payments from a project to the provider of funds. What the project lost (interest) has become a gain to the provider of funds.

In effect, both figures are equal and cancel out without any net increase to society of funds. Therefore in economic analysis, interest charges are excluded since they only represent transfer payments.

The second transfer payment we shall consider is tax. When a project is profitable it is expected to pay taxes to the government at the ruling rate. In computing the profit of a project taxes are deducted to arrive at net profit. Taxes therefore appear as outgoing cash flows. Taxes represent transfer payments from a project to government.

In the economic analysis of a project, taxes are excluded because from the point of view of the society, they are only a transfer of resources from one section of the economy to another.

The third transfer payment is subsidies. In a traditional private sector setting, it would be unheard of to talk of subsidies. But in economic analysis, subsidies appear as important data. Most public sector projects enjoy government subsidies to enable the poor gain access to certain

services which ordinarily they cannot afford without government assistance. Subsidies represent opportunity costs to a nation as a whole. Therefore in estimating the true cost of a project in an economic analysis, subsidies should be included.

3.4 Linkage Effects of a Project

Consider a simple case where a university is newly located in an environment. Many investments will begin to spring up. New housing developments will begin to spring up; canteens will begin to spring up; hair dressing salons, etc. will begin to spring up to cater for the needs of the new university community. Such constitute the linkage effects of a project.

Generally, there are two types of linkage effects which we shall briefly discuss:

Forward Linkage Effects

Forward linkage is the stimulus given to industries that use the products of a project. A case in point is a flour manufacturing project. Flour has so many uses. If a flour mill is located in an environment, it will lead to the establishment of such projects as bakeries which will use the flour.

Backward Linkage Effects

Backward linkage demonstrates the stimulus to industries that supply the inputs to a project. For example, the establishment of a flour mill in an environment will lead to demand for wheat which is a major input for flour mill. The flour mill will lead to investment in wheat cultivation.

Also, the establishment of a car assembly plant will lead to the establishment of tyre manufacturing plants that need to supply tyres to the car assembly plant.

Example of an Economic Analysis

In the year 2006, the World Bank was considering the desirability or otherwise of assisting Nigeria set up an ethanol plant covering thousands of hectares in the Niger Delta area.

Under the scheme, young farmers will be allocated hectares of land for subsidised cassava cultivation. Such inputs like fertilisers will be heavily subsidized while technical advice will be provided by the World Bank/Nigerian agricultural experts.

4.0 CONCLUSION

In this unit, we discussed the nature of economic analysis and compared it with the financial analysis of a project. We discussed net present worth, benefit cost ratio and the internal rate of return (IRR).

5.0 SUMMARY

Introduction to economic analysis has provided us with the tools to conduct economic analyses, with financial analyses as a starting point. Financial analysis is the private sector's view of a project without considering a project's impact on the society. Economic analysis is a macro view of a project, taking into consideration the project's impact on society.

6.0 TUTOR-MARKED ASSIGNMENT

What do you see as the basic differences between the financial analysis of a project and the economic analysis of a project?

7.0 REFERENCES/FURTHER READINGS

Leon Ikpe (1999). *Project Analysis and Evaluation*. Lagos: Impressed Publishers.