

COURSE GUIDE

ECO 103 INTRODUCTORY MATHEMATICS I

Course Team

Dr Chukwunonso Francis Onoh (PhD)
(Course Writer) - NOUN
Dr Emmanuel I. Ajudua (Course Editor) -
NOUN
Sunday Mathias (Instructional Designer)-
NOUN
Mrs Mercy Dal (Typesetter/Formatter) –
NOUN



NATIONAL OPEN UNIVERSITY OF NIGERIA

© 2026 by NOUN Press
National Open University of Nigeria
Headquarters
University Village
Plot 91, Cadastral Zone
Nnamdi Azikiwe Expressway
Jabi, Abuja

Lagos Office
14/16 Ahmadu Bello Way
Victoria Island, Lagos

E-mail : centralinfo@nou.edu.ng
URL: www.nou.edu.ng

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

Printed 2026

ISBN: 978-978-786-741-9

CONTENTS

Introduction.....	iv
Course Competencies.....	iv
Course Objectives.....	iv
Working Through This Course.....	v
Study Units.....	v
References and Further Readings.....	v
Presentation Schedule.....	vi
Assessment.....	vi
Assignments & Grading.....	vi
How to get the Most from the Course.....	vii

INTRODUCTION

Introductory Mathematics for Economists I is a two-unit course for 100-level students at the National Open University of Nigeria. It comprises 11 study units, subdivided into 3 modules. The materials have been developed with the Nigerian context in mind, using simple, local examples. This course guide gives you an overview of the course. It also provides you with the organisation and the course requirements.

COURSE COMPETENCIES

These lecture notes, in general, gave students in undergraduate programs in economics, business, finance, and social science an understanding of mathematical reasoning, fundamental concepts in mathematical economics, and an extended mathematical toolbox. The only prerequisite to follow the topics covered in this lecture note is patience. Mathematics has become the language of economics, business, and finance. Most undergraduate social science programs use geometric tools to explain relationships and theories. However, the inherent limitations of geometric tools call for the use of general algebra. Thus, it necessitates training in the use of mathematics techniques and tools. From experience, large numbers of undergraduate students of social science have “math-phobia” and “math-aversion” due to the unclear presentation of the course. These lecture notes have been developed with the Nigerian context in mind, using simple, local examples to help replace feelings of phobia and aversion with passion and appreciation for the subject matter, and to foster understanding and application of mathematical techniques and tools to real-life situations.

COURSE OBJECTIVES

The main objective of this course is to provide students with an understanding of mathematical concepts, techniques, tools, and their applications in real-life situations. And the course has its specific objective to;

- give an understanding of the stages and processes involved in identifying business problems;
- give an understanding of what constitutes business and economics problems;
- help students relate mathematics techniques and tools to different business and economics problem analysis; and
- generate, access and work with quantitative ideas in solving economic problems.

WORKING THROUGH THIS COURSE

To complete this course, you are required to go through the study units in this lecture note and other related materials. You will also need to undertake practical exercises, for which you will need a pen, a notebook, and other materials listed in this guide. The exercises are to help you understand the basic concepts and principles in the lecture notes. At the end of each unit, you will be required to submit written assignments for assessment purposes. At the end of the course, you will write a final examination.

STUDY UNITS

There are 3 modules in this course, broken into 11 study units.

Module 1 Set Theory, Factors and Logarithms

Unit 1	Set Theory
Unit 2	Factors and Experiments
Unit 3	Logarithms

Module 2

Unit 1	Equations, Functions, and Change of Subject of Formulae
Unit 2	Linear Equations and Linear Simultaneous Equation
Unit 3	Quadratic Equation
Unit 4	Simultaneous Linear and Quadratic Equations

Module 3

Unit 1	Trigonometry
Unit 2	Coordinate Geometry
Unit 3	Inequalities
Unit 4	Differentiation

REFERENCES AND FURTHER READINGS

- Fabayo J.A (1996) Mathematical analysis in Economics: Obafemi Awolowo University Press Limited. Ile-Ife, Nigeria
- Agbadudu A.B. (1998). Mathematical Methods in Business and Economics: United City Press, Benin City, Nigeria.
- Eke, C.E. (2010). Solution-Oriented on Business Mathematics: Justice Jeco Printing & Publishing Global (recommended).
- Adibe, O.R. (1995). Introduction to Management Mathematics for Business students: Model publishing Ltd Aba.

PRESENTATION SCHEDULE

1. Week 1-2: Set Theory
2. Week 3-4: Factors and Experiments
3. Week 5-6: Logarithms
4. Week 5-7: Trigonometric Functions
5. Week 8: Equation Functions
6. Week 9: Coordinate Geometry
7. Week 10: Inequalities
8. Week 11: Introduction to Differentiation
9. Week 12: Revision

The presentation schedule in your course materials includes the important dates for completing tutor-mark assignments and attending tutorials this year. Remember, you are required to submit all your assignments by the due date. You should guard against falling behind in your work.

Set theory, relations, algebraic operations, and economic applications of various types of algebraic functions; and their applications in economics; matrix algebra and special matrices; introduction to calculus of algebraic functions of a single variable with applications to marginal analysis; and optimisation in economics.

ASSESSMENT

The assessments for this course are of two types: the tutor-marked assignments and the written examination.

In attempting the assignments, you are expected to apply information, knowledge and techniques gathered during the course. The assignments must be submitted to your tutor for formal assessment in accordance with the deadlines stated in the Presentation Schedule and the Assignments File. The work you submit to your tutor for assessment will count for 30 % of your total course mark. At the end of the course, you will need to sit for a final written examination of three hours' duration. This examination will also count for 70% of your total course mark.

ASSIGNMENTS & GRADING

Academic Honesty: All class work should be done independently, unless explicitly stated otherwise on the assignment.

You may discuss general solution strategies, but must write up the solutions yourself.

If you discuss any problem with anyone else, you must write their name at the top of your assignment, labelling them “collaborators”.

HOW TO GET THE MOST FROM THE COURSE

In open distance learning (ODL), the study units replace the university classroom lectures. This is one of the merits of distance learning; you can read and work through the outlined study materials at your own pace, time and place of your choice. The idea is that you are reading the lecture notes rather than listening to them. In the same way that a lecturer might give you some reading to do, the study units contain instructions on when to read your set of books or other materials and practice some practical questions. Just as a lecturer might give you an in-class exercise or quiz, your study units provide exercises for you to do at an appropriate point in time. Each study unit follows a common format. The following is a practical strategy for working through the course. Always remember that your tutor's job is to help you. When you need his assistance, do not hesitate to call and ask your tutor to provide it. Follow the under-listed pieces of advice fully: -

- 1) Organise a Study Schedule: refer to the course overview for more details. Note the time you are expected to spend on each unit and how the assignments relate to the units.
- 2) Having created your personal study schedule, ensure you adhere strictly to it. The major reason students fail is their inability to work with their study schedule, which leads to getting behind in their coursework. If you have difficulties working with your schedule, you must let your tutor know.
- 3) Assemble the study material. Information about what you need for a unit is given in the 'overview' at the beginning of each unit. You will almost always need both the study unit you are working on and one of your set books on your desk at the same time.
- 4) Work through the study unit. The unit's content has been arranged to provide a sequence for you to follow. As you work through the unit, you will be instructed to read sections from your set books or other articles. Use the unit to guide your reading.
- 5) Review the objectives for each unit to be informed that you have achieved them. If you feel uncertain about any of the objectives, review the study material or consult your tutor.
- 6) When you are sure that you have achieved the objectives of a unit, you can then start on the next unit. Proceed unit by unit through the course and try to space your studies so you stay on schedule.
- 7) When you have submitted an assignment to your tutor for marking, do not wait for its return before starting on the next unit. Keep to your schedule. You are strongly advised to consult your tutor as soon as possible if you have any challenges or questions.
- 8) After completing the last unit, review the course and prepare yourself for the final examination. Check that you have achieved the objectives of the units (listed at the beginning of each unit) and the course objective (listed in this Course Guide).

- 9) After completing the last unit, review the course and prepare yourself for the final examination. Check that you have achieved the objectives of the units (listed at the beginning of each unit) and the course objective (listed in this Course Guide).
- 10) Keep in touch with your study centre. Up-to-date course information will be constantly made available to you there.

Online Facilitation

There will be ten hours of tutorials to support this course. The dates and times of these tutorials will be communicated together with the name and phone number of your tutor as soon as you are allocated a tutorial group. You should call your tutor to discuss related assignments, progress, and any difficulties you might encounter to assist you during the course. You must mail your completed assignment to your tutor well before the due date (at least 2 working days' notice is required). The assignments will be marked by your tutor and returned to you as soon as possible. Do not hesitate to contact your tutor by telephone or e-mail for personal discussions if you need help. The following might be circumstances in which you would find help necessary:

- i. You do not understand any part of the study unit
- ii. You have difficulty/difficulties with the self-assessment exercise(s)
- iii. You have a question or problem with your tutor's comments on any assignment or with the grading of an assignment.

You are advised to attend tutorials regularly. This is the only opportunity to have a face-to-face contact with your tutor and ask questions. You can report any problems encountered during the course of study. To gain the maximum benefit from course tutorials, prepare a question list before attending them, participate actively, and ensure you ask questions.

CONTENTS	PAGE
Module 1 Set Theory, Factors and Logarithms.....	1
Unit 1 Set Theory.....	1
Unit 2 Factors and Experiments.....	32
Unit 3 Logarithms.....	47
Module 2 Equations and Formulae.....	58
Unit 1 Equations, Functions, and Change of Subject of Formulae.....	58
Unit 2 Linear Equations and Linear Simultaneous Equation..	71
Unit 3 Quadratic Equation.....	89
Unit 4 Simultaneous Linear and Quadratic Equations.....	106
Module 3 Trigonometry, Coordinate Geometry, Inequalities and Introduction to Differentiation.....	115
Unit 1 Trigonometry.....	115
Unit 2 Coordinate Geometry.....	126
Unit 3 Inequalities.....	148
Unit 4 Differentiation.....	166

MODULE 1 SET THEORY, FACTORS AND LOGARITHMS

Unit 1	Set Theory
Unit 2	Factors and Experiments
Unit 3	Logarithms

Unit 1 Set Theory

Unit Structure

- 1.1 Introduction
- 1.2 Learning Outcomes
- 1.3 Meaning and Classification of Sets
 - 1.3.1 The empty set
 - 1.3.2 Subsets
 - 1.3.3 Equality of sets
 - 1.3.4 The universal set
- 1.4 Laws of Set Operation
- 1.5 Venn Diagrams
- 1.6 Application of Sets to Managerial and Economic Problems
- 1.7 Summary
- 1.8 References/Further Readings
- 1.9 Possible Answer to Self-Assessment Exercised



1.1 Introduction

You are Welcome to the ultimate organising system of the deep learning world. Whilst it is easy to get swept up in the glamour of training massive neural networks, every algorithm relies on a quiet, foundational logic to categorise and manipulate data. This unit introduces the absolute starting point of that logic, the fundamental notion of grouping, which serves as the invisible scaffolding for everything from splitting datasets to defining multi-class probability spaces. By understanding how we establish these boundaries, you will see how chaotic, raw information is transformed into structured collections that our models can actually process.

As you explore this section, try to look past the formal notation and focus on the spatial intuition instead. We will guide you through how these collections interact and overlap, helping you visualise the mathematical structure beneath the algorithms. Keep this perspective close in the chapters ahead, as these simple rules of membership will directly shape how you construct loss functions and partition high-dimensional spaces later in the course.



1.2 Learning Outcomes

By the end of this unit, you should be able to:

- define the notion of a set;
- explain set notations and terminologies; and
- apply sets to managerial and economic problems.



1.3 Meaning and classification of a Set

The collection of any defined objects is called a set. That is, the possibility of determining readily whether or not a given object belongs to a collection. Any object in a given set is called an element or member. A set is stated by capital letters, and elements or members in a set are stated by small letters. So, x is an element in the set A , the equation is written as $x \in A$, and if y is not an element of set A , the equation is written as $y \notin A$. The number of elements in set A is stated by $n(A)$. There are two classes of sets: finite sets and infinite sets. A finite set has a definite number of elements, while an infinite set has no definite number of elements; it does not end.

A set defined by listing its elements;

$$A = \{ 2, 4, 6, 8 \}$$

That is, set A has 2, 4, 6, and 8 as its elements or members. Again, a set can be defined by stating the properties of its elements. like set $A = \{ 2, 4, 6, 8 \}$ can be written $A = \{ 2n: 1 \leq n \leq 4, \text{ and } n \text{ is an integer} \}$.

1.3.1 The empty set

There is a time when a set contains no elements. Like, the set of triangles with four sides contains no elements, since there is no triangle with four sides. A set without an element is called the *empty* or *null* set, denoted by ϕ .

1.3.2 Subsets

If all elements in set A are also the elements in set B , it can be stated that set A is a *subset* of set B or that A is contained in B . The equation is written as $A \subseteq B$ or $B \supset A$. The equation is written as $\{ a, b \} \subseteq \{ a, b, c \}$ since each of the two elements in $\{ a, b \}$ belongs to $\{ a, b, c \}$. From this definition, all sets are subsets of themselves; that is, for all $A, A \subseteq A$. Any subset of a set that is not the set itself is called a *proper* subset of that set. (Sometimes the symbol $\subsetneq$ is used for a

subset and the symbol $\subset$ for a real subset). An empty set, by definition, is a subset of every set. The subsets of $\{a, b\}$ are $\{a, b\}$, $\{a\}$, $\{b\}$ and $\emptyset$. It can be shown that a set of n elements has 2^n subsets. Note: There is a need for care in distinguishing between the elements of a set and its subsets. For example, if $C = \{a, p\}$, then a is an element of C ; *again*, $\{a\}$ is a set that has a as an element. So, it can be written $a \in C$, and $\{a\} \subset C$.

1.3.3 Equality of sets

Two sets are said to be equal when they contain the same elements. That is, set A is equal to set B , it is written as $A = B$, and when set T is not equal to set S , it is written as $T \neq S$. Therefore, $\{2, 3\} = \{3, 2\}$ and $\{2, 3\} \neq \{2, 4\}$.

Note: the order of arrangement or listing of elements in a set does not matter. So, can take from $B = D \Leftrightarrow B \subset D$ and $D \subset B$. This provide a way to show that two sets are equal. So, all empty sets are equal: for empty sets to be unequal, one of the sets needs to contain an element, which is impossible since the sets are empty sets without any element.

1.3.4 The universal set

A set that contains all objects or elements, and all the other sets are [subsets](#), is called a universal set. The universal set is denoted by U ; it contains all the elements of the related sets, and the elements of the related sets are repeated. Where C and D are two sets, such that C has 1, 2, 3 as elements and D has 1, a, b, c as elements, and the universal set is given as $U = \{1, 2, 3, a, b, c\}$. Note: for any set, it is assumed that there is a universal set, all sets and considerations are subsets of U . Again, different sets may have different universal sets. So, there is a need to restrict the elements to those relevant to the problem under study. That is, the universal set contains all relevant elements. If the need is to study the average score of English students in Nigerian secondary schools, the universal set is all students in secondary schools, whether junior or senior.

1.4 Laws of Set Operations

(a) Idempotent Properties: This property states that the union and intersection of the same set give the set.

$$A \cup A = A$$

$$A \cap A = A$$

For example, given that $A = \{1, 2, 3, 4, 5, 6\}$

$$B = \{1, 2, 5, 7, 10\}$$

$$A \cup A = \{1, 2, 3, 4, 5, 6\} \cup \{1, 2, 3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6\} = A$$

Likewise, $B \cap B = \{1, 2, 5, 7, 10\} \cap \{1, 2, 5, 7, 10\}$
 $B \cap B = \{1, 2, 5, 7, 10\} = B$

(b) Commutative Properties: The commutative law states that $A \cap B = B \cap A$ and $A \cup B = B \cup A$.

This implies that the order of the set does not matter in their union or intersection.

For example:

If $A = \{1, 2, 3, 4, 5, 6\}$ and

$$B = \{3, 4, 5, 7, 10\}$$

Then,

$$A \cap B = \{3, 4, 5\}$$

$$\text{Likewise, } B \cap A = \{3, 4, 5\}$$

$$\text{Hence, } A \cap B = B \cap A = \{3, 4, 5\}$$

$$\text{Similarly, } A \cup B = B \cup A$$

$$A = \{1, 2, 3, 4, 5, 6\}, \quad B = \{3, 4, 5, 7, 10\}$$

$$\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 7, 10\} \text{ and}$$

$$B \cup A = \{1, 2, 3, 4, 5, 6, 7, 10\}$$

$$\text{Hence, } A \cup B = B \cup A \text{ and } A \cap B = B \cap A$$

(c) Associative Properties: This states that

$$A \cup (B \cap C) = (A \cup B) \cap C \text{ and } A \cap (B \cup C) = (A \cap B) \cup C$$

For example,

$$\text{let } A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{2, 4, 6, 8, 10\}, C = \{3, 6, 9, 10\}$$

$$\text{Then, } B \cap C = \{2, 3, 4, 6, 8, 9, 10\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cup \{2, 3, 4, 6, 8, 9, 10\}$$

$$= \{1, 2, 3, 4, 5, 6, 8, 9, 10\}$$

Likewise,

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$C = \{3, 6, 9, 10\}$$

$$\therefore (A \cup B) \cap C = \{1, 2, 3, 4, 5, 6, 8, 9, 10\}$$

$$\therefore A \cup (B \cap C) = (A \cup B) \cap C$$

In the same manner,

$$A \cap (B \cup C) = (A \cap B) \cup C$$

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B \cap C = \{6, 10\}$$

.

$$\therefore A \cap (B \cup C) = \{6\}$$

$$A \cap B = \{2, 4, 6\}$$

$$C = \{3, 6, 9, 10\}$$

$$\therefore (A \cap B) \cup C = \{6\}$$

$$\text{Hence, } (A \cap B) \cup C = \{6\} = A \cap (B \cup C) = \{6\}$$

(d) Distributive Law: This property states that
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

From the example above, $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{2, 4, 6, 8, 10\}$

$C = \{3, 6, 9, 10\}$

$A \cap C = \{6, 10\}$

$A = \{1, 2, 3, 4, 5, 6\}$

$\therefore A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 10\}$

But $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$

$A \cap C = \{1, 2, 3, 4, 5, 6, 9, 10\}$

$(A \cup B) \cap (A \cap C) = \{1, 2, 3, 4, 5, 6, 10\}$

Hence,

$A \cup (B \cap C) = (A \cup B) \cap (A \cap C) = \{1, 2, 3, 4, 5, 6, 10\}$

Likewise, we can prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$B \cup C = \{2, 3, 6, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5, 6\}$

$A \cap (B \cup C) = \{2, 3, 6\}$

$A \cap C = \{3\}$, $(A \cap B) = \{2, 6\}$

$\therefore (A \cap B) \cup (A \cap C) = \{2, 3, 6\}$

Hence, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{2, 3, 6\}$

(e) Identity Properties: This states that the union or intersection of a given set with respect to an empty set gives the setback.

For instance, $A \cup \emptyset = A$, $A \cap \emptyset = \emptyset$ If $A = \{1, 2, 3, 4, 5, 6\}$ $\emptyset = \{ \}$

$\therefore A \cup \emptyset = \{1, 2, 3, 4, 5, 6\} = A$

$A \cap \emptyset = \{1, 2, 3, 4, 5, 6\} = \emptyset$

Similarly, the union of a given set with respect to the universal set gives the universal set, while the intersection of a given set with respect to the universal set gives the setback i.e., $A \cup \mu = \mu$ and $A \cap \mu = A$

E.g. if $A = \{1, 2, 3, 4, 5\}$

$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Then $A \cup \mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = \mu$

$A \cap \mu = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\therefore A \cap \mu = \{1, 2, 3, 4, 5\} = A$

(f) Complement Properties: This states that

$A \cup A^1 = \mu$

$A \cap A^1 = \emptyset$ or $\{ \}$ or empty set $(A^1)^1 = A$

$\mu = \emptyset$

For example,

$A = \{1, 2, 3, 4, 5\}$

$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\therefore A^1 = \{6, 7, 8, 9, 10\}$

$\therefore A \cup A^1 = \{1, 2, 3, 4, 5\} \cup \{6, 7, 8, 9, 10\} = \mu$

$A \cap A^1 = \{1, 2, 3, 4, 5\} \cap \{6, 7, 8, 9, 10\} = \emptyset$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = \mu$$

$$A^1 = \{6, 7, 8, 9, 10\}$$

$$\therefore (A^1)^1 = \{1, 2, 3, 4, 5\} = A$$

$$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\mu^1 = \{ \} \text{ or } \emptyset \text{ (Empty set)}$$

(g) De Morgan's Law This states that $(A \cup B)^1 = A^1 \cap B^1$ and that $(A \cap B)^1 = A^1 \cup B^1$

For example, $\mu = \{1, 2, 3 \dots 10\}$

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{4, 6, 7, 8\}$$

Then

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$(A \cup B)^1 = \{9, 10\}$$

$$A^1 = \{7, 8, 9, 10\}, B^1 = \{1, 2, 3, 5, 9, 10\}$$

$$A^1 \cap B^1 = \{9, 10\}$$

$$\text{Hence } (A \cup B)^1 = A^1 \cap B^1$$

$$\text{Similarly, } (A \cap B)^1 = A^1 \cup B^1$$

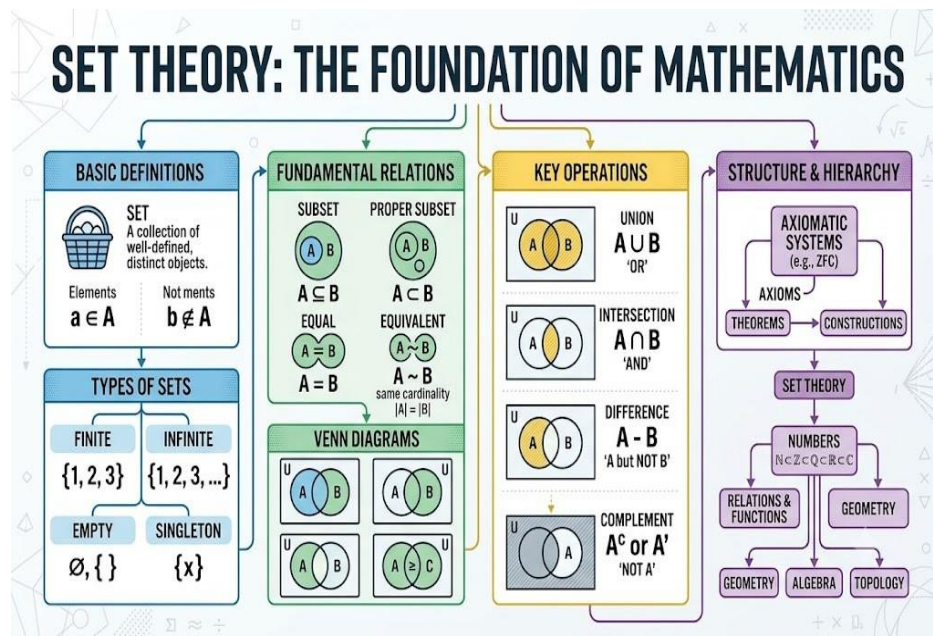
$$\text{From the example above } A \cap B = \{4, 6\}$$

$$(A \cap B)^1 = \{1, 2, 3, 5, 7, 8, 9, 10\}$$

$$A^1 = \{7, 8, 9, 10\}, B^1 = \{1, 2, 3, 5, 9, 10\}$$

$$A^1 \cup B^1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

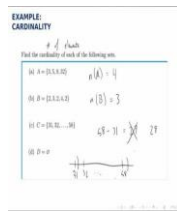
$$\text{Hence } (A \cap B)^1 = A^1 \cup B^1$$



Source AI generated on 16 07 2026 Adopted by mathias

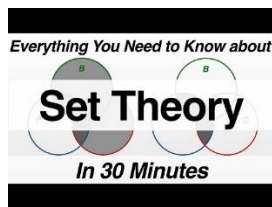
Click this link for more understanding

<https://www.youtube.com/watch?v=Rh8ss4TBAdo>



Click this link for more understanding

<https://www.youtube.com/watch?v=5ZhNmKb-dqk>



Self-Assessment Exercise 1

i. Which of the following statement(s) is/are valid?

- (a) $A \cup A = A$
- (b) $A \cap A = A$
- (c) $A \cup \emptyset = A$
- (d) $A \cup \mu = U$
- (e) $A \cap \emptyset = \emptyset$
- (f) $A \cap \mu = A$
- (g) The complement of A is A (i.e. $(A^c)^c = A$)
- (h) $(B \cap C) = B \cap C$
- (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- (j) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii. Given $A = \{4, 5, 6\}$, $B = \{3, 4, 6, 7\}$ and $C = \{2, 3, 6\}$, verify:

- (a) Associative law
- (b) Distributive law
- (c) De Morgan's law

1.5 Venn diagrams

A Venn diagram is used to show the relationships among objects or finite groups of things. Venn diagrams visually show the similarities and differences between two or more objects or elements. It is a useful educational tool for illustrating the elements of the universal set.

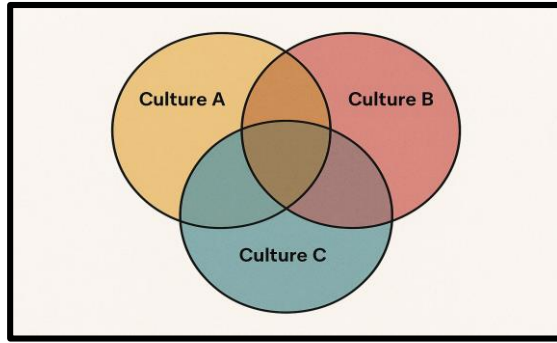


Fig. 1.1 Venn diagram (universal Set)

Self-Assessment Exercise 2

- i. What is a Venn diagram? What is its relevance to set theory?
- ii. Draw a Venn diagram illustrating the relationship between sets X and Y if:
 - (i) Some elements are common to X and Y
 - (ii) No element is common to X and Y
 - (iii) Y contains every element in X.

Simple Set Problems Involving Venn Diagrams. A Venn diagram applies to all problems involving set operations. However, it is very important to commence its application to simple set problems involving the use of numbers to indicate the elements or membership of a set.

Example 1

If $A = \{\text{Prime factors of } 30\}$

$B = \{\text{Prime factors of } 70\}$

$C = \{\text{the prime factors of } 42\}$

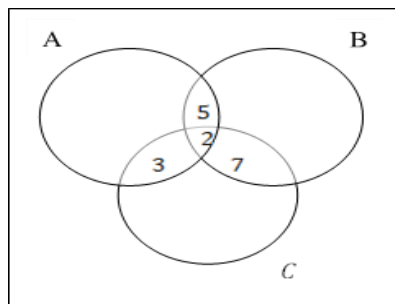
- (a) List the elements of A, B and C
- (b) Show their relationship in a Venn diagram

Solution

(a) $A = \{2, 3, 5\}$

$B = \{2, 5, 7\}$

$C = \{2, 3, 7\}$



It should be noted that '2' is a common element to all three sets, so it takes the centre stage, i.e., a point of intersection of A, B and C '5' is common to only A and C. The only number common to sets B and C is 7. Having considered the sets common to two or more sets, we observe that each of the three sets is left with its own independent space. Hence, the spaces were left vacant. It should also be noted that the universal set is not given, so there is no element found outside the three oval shapes representing sets A, B and C.

Example 2

Given the universal set as $\mu = \{1, 2, 3, 4 \dots 10\}$

$A = \{2, 3, 4, 5, 6\}$

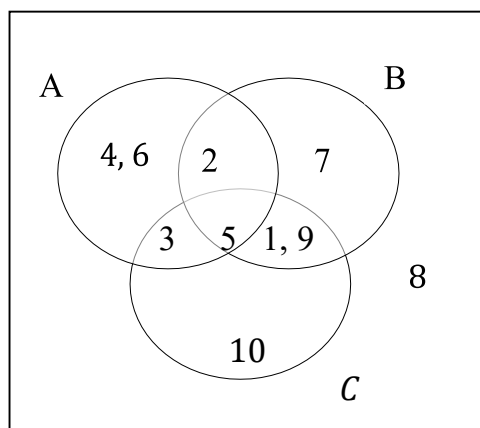
$B = \{1, 2, 5, 7, 9\}$

$C = \{1, 3, 5, 9, 10\}$

Required:

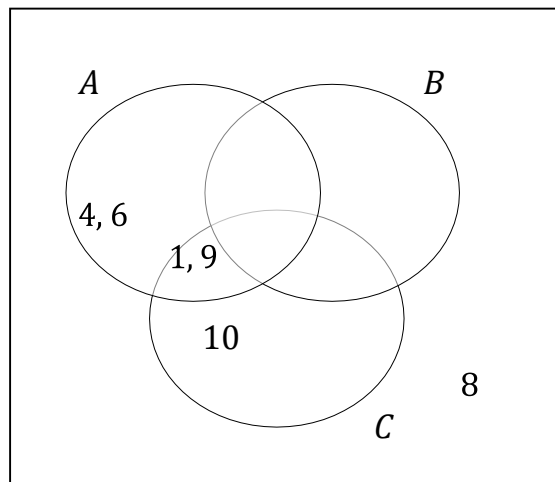
- (a) Present the information using a Venn diagram.
- (b) On different Venn diagrams, shade the region that satisfies the following
 - (i) $A \cap B \cap C$
 - (ii) B^1
 - (iii) $(B \cup C)^1$
 - (iv) $A \cup B \cup C$
 - (v) $(A \cap B \cap C)^1$
- (c) Use the diagram in (b) above to list the elements of sets obtained from b (i)-(v) above.

Solution



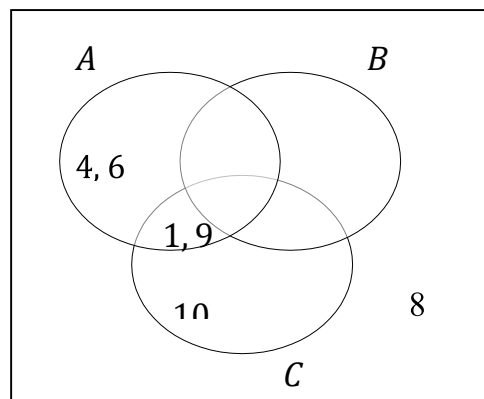
Note: '8' is an element of the universal set but not present in any of the sets A, B and C. So, it is written outside the three oval shapes representing sets A, B, and C.

b. (i) $A \cap B \cap C$



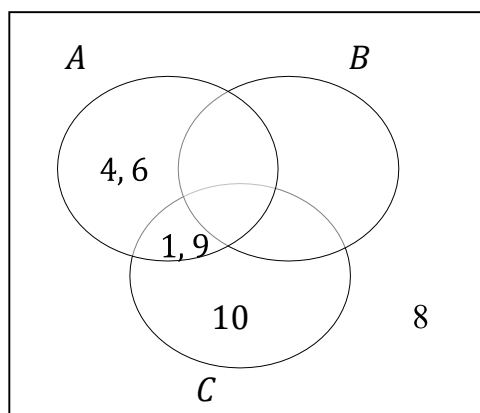
Note: The shaded area represents the region of intersection of A, B and C, i.e. elements common to the three sets.

(ii) B^1

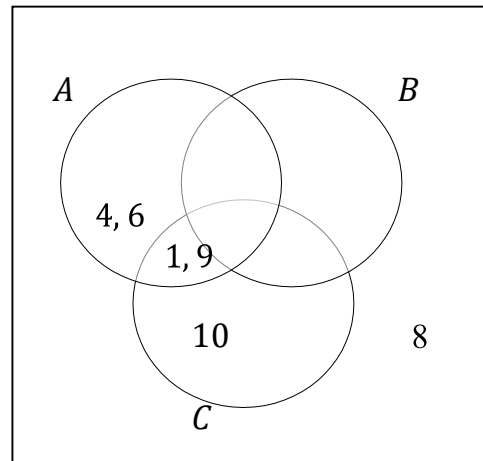


Note: B^1 is represented by the elements found outside set B, i.e., all the areas in set B are shaded.

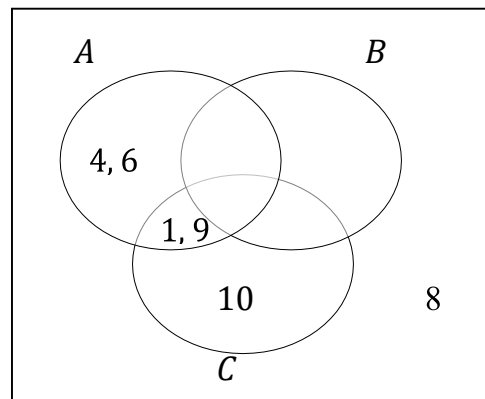
(iii) $(B \cup C)^1$



Note: $(B \cup C)^1$ implies the area outside the enclosure of B and C. So, all the spaces outside $(B \cup C)$ are shaded.

(iv) $A \cup B \cup C$ 

Note: $(A \cup B \cup C)$ is the totality of elements in sets A, B and C. Hence, all the set spaces are shaded.

(v) $(A \cap B \cap C)^1$ 

Note: $(A \cap B \cap C)^1$ implies the area outside $A \cap B \cap C$. Hence, the entire space is shaded, except for $A \cap B \cap C$.

- C (i) $A \cap B \cap C = \{5\}$
(ii) $B^1 = \{3, 4, 6, 8, 10\}$
(iii) $(B \cup C)^1 = \{4, 6, 8\}$
(iv) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 9, 10\}$
(v) $(A \cap B \cap C)^1 = \{1, 2, 3, 4, 6, 7, 9, 10\}$

Self-Assessment Exercise 3

i. Given that

$A = \{\text{all prime numbers between 10 and 30}\}$

$B = \{\text{all even numbers between 9 and 29}\}$

$C = \{\text{all multiples of 3 between 10 and 29}\}$

(a) List the elements of each set and draw a Venn diagram showing the Required: relationship among the sets.

(b) In separate Venn diagrams, show the region that satisfies each of

(i) $A \cup (B \cap C)$

- (ii) $(A \cap B) \cup C$
 - (c) List the elements of b(i) and b(ii) above
 - (d) Is $A \cup (B \cap C) = (A \cap B) \cup C$? What do you think is responsible for this?
- ii. If P and Q are sets, using a Venn diagram, shade the following relationship on different diagrams
- a) $P \cap Q$
 - b) $P \cup Q$
 - c) P^1
 - d) Q^1

This unit introduces us to the origin of the Venn diagram, its relevance in set theory and analysis, and its practical application in presenting elements of sets and their interrelationships, as well as in solving problems involving set notation. Representation of sets and their elements with the use of a diagram (Venn diagram) is not only meant to show the interaction among the elements or among the sets, but it is also an alternative way of solving problems involving set notations. This is illustrated in Example 2 of this unit.

1.6 Application of Set Theory to Managerial and Economic Problems

In our day-to-day activities, we are confronted with categorising people or objects into different classes or divisions. In the course of doing this, there may be an overlap of one element appearing in two or more sets. For instance, students in a class may be able to speak English or French fluently. Among this set of students, some may be able to speak both languages, and some may not be able to speak either fluently. In analysing this, there may be a need to introduce a Venn diagram.

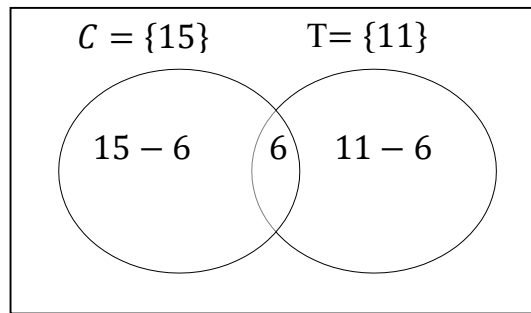
Another example is the elective courses taken by students in higher institutions. For example, if courses A, B, and C are made available as electives. Some students may offer one of the courses; some may offer a combination of two courses; some may offer a combination of two courses; and some may offer all three courses. It is equally possible that some students do not offer any of the three courses. To carry out mathematical analyses of the distribution of students who offer courses and course combinations, an appropriate Venn diagram may be required.

Example 1

A survey of a class shows that 15 pupils play cricket, 11 play tennis, and 6 play both cricket and tennis. How many pupils are there in the class, if everyone plays at least one of these games?

Solution

$$\mu = x$$



$n(C) = 15$; No of Pupils that play Cricket

$n(T) = 11$; No of Pupils that play Tennis.

Note: If 6 pupils play both games, recall that the pupils are part of the entire pupil group that plays cricket, so the remaining pupils who play cricket but not tennis equals $15 - 6 = 9$. Likewise, for Tennis, 11 of the 11 students are already among the 6 pupils who play both games. Therefore, those who play tennis alone equals $11 - 6 = 5$.

Since everybody plays at least one of these games, it implies that there are no pupils who play neither of the two games.

$\therefore$ Number of students in the class = $6 + 9 + 5 = 20$ pupils

$\therefore x = n(C \cup T) = 20$

Example 2

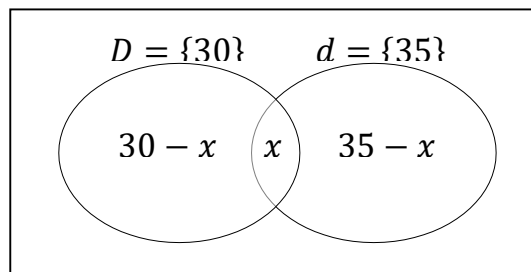
A survey conducted among 50 members of staff in an organisation shows that 30 have degrees and 35 have diplomas, with 8 employees possessing neither certificate.

Required

- (a) How many have a degree but not a diploma
- (b) How many have both a Degree and a Diploma

Solution

$$\mu = 50$$



“D” represents the employees who have a degree

“d” represents the employees who have a diploma

“x” represents the employees who have both a degree and a diploma

μ = Universal set or set of all employees

$$\therefore x = n(D \cap d)$$

$$\therefore 30 - x + x + 35 - x = 50 \quad 30 + 35 - x = 50$$

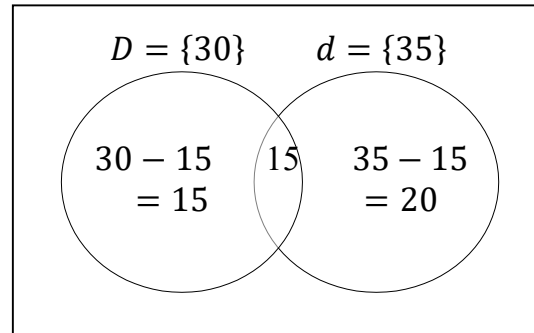
$$65 - x = 50$$

$$65 - 50 + x$$

$$x = 15$$

The Venn diagram can now be redrawn as follows:

$$\mu =$$



$$50$$

$$\text{Note: } 15 + 15 + 20 = \mu$$

$$\mu = 50$$

(a) No of employees who have a degree but not a diploma $= 30 - x = 30 - 15 = 15$

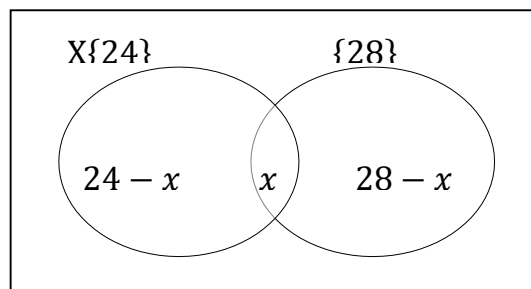
(b) No of employees who have both a degree and a diploma $x = 15$

Example 3

Given that $n(X) = 24$, $n(Y) = 28$, $n(X \cup Y) = 40$, $n(X \cap Y) = 5$. Use a Venn diagram to find

- (i) $n(X \cap Y)$
- (ii) $n(X)$
- (iii) $n(X \cap Y)$
- (iv) μ

Solution



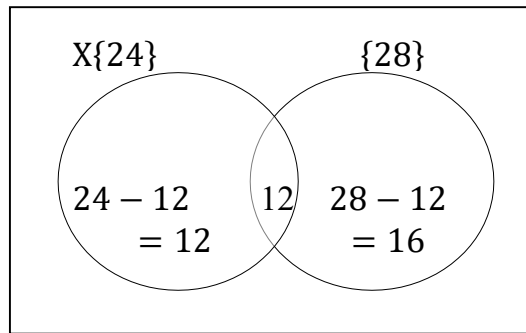
$$\text{Note: } n(X \cup Y) = 24 - x + x + 28 - x = 40$$

$$52 - x = 40$$

$$52 - 40 = x$$

$$x = 12$$

The Venn diagram can now be redrawn as



From the Venn diagram

(i) $n(X \cap Y) = 12$

(ii) $n(X^c) = 16 + 5 = 21$

Note: $n(X^c)$ represents values outside the oval shape of X, i.e. 16 and 5.

(iii) $n(X \cup Y)^c = 12 + 16 + 5 = 33$

Note: $n(X \cup Y)^c$ represents the values outside the intersection of X and Y.

These values are 12, 5, and 16.

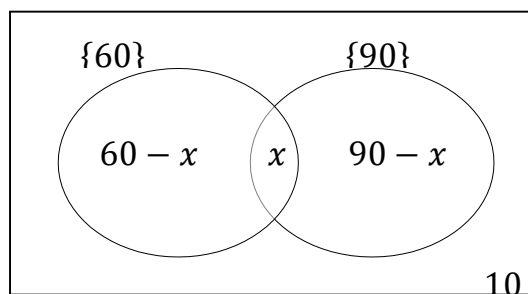
(iv) $\mu = 12 + 12 + 16 + 5 = 45$

Note: μ is the totality of all the values in the Venn diagram.

Example 4

In a school of 140 pupils, 60 have beans for breakfast, and 90 have bread. How many students have both beans and bread for breakfast if 10 pupils took neither beans nor bread?

$$\mu =$$



$$140$$

$$n(B) = \text{no of pupils that take beans} = 60$$

$$n(b) = \text{no of pupils that take bread} = 90$$

x = no of pupils that have both beans and bread for breakfast.

$$60 - x + x + 90 - 10 = 140$$

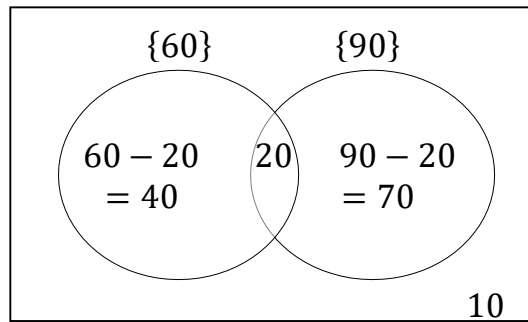
$$160 - x = 140$$

$$160 - 140 = x$$

$$20 = x$$

$$x = 20$$

The Venn diagram is redrawn as follows:



Note: $10 + 40 + 20 + 70 = 140$

$$40 + 20 = 60$$

$$20 + 70 = 90$$

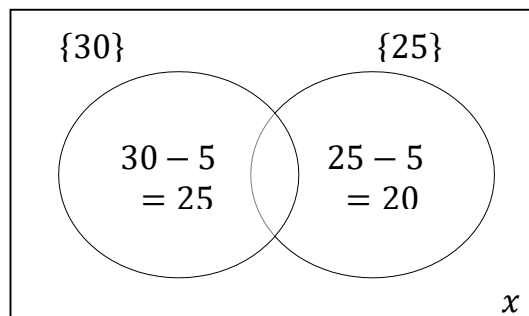
$\therefore$ 20 pupils have both beans and bread for breakfast

Example 5

In a class of 60 students, 30 speak English fluently, while 25 speak French fluently. If 5 students can speak both languages, how many cannot speak either of the languages? How many speak only one language?

Solution

$$\mu = 60$$



$n(E)$ = No of students who speak the English language fluently = 30

$n(F)$ = No of students who speak French fluently = 25

$n(EnF)$ = no of students who can speak both English and French = 5

x = No of students who cannot speak any of the Languages

$$\therefore x + 25 + 5 + 20 = 60$$

$$x + 50 = 60$$

$$x = 60 - 50$$

$$x = 10$$

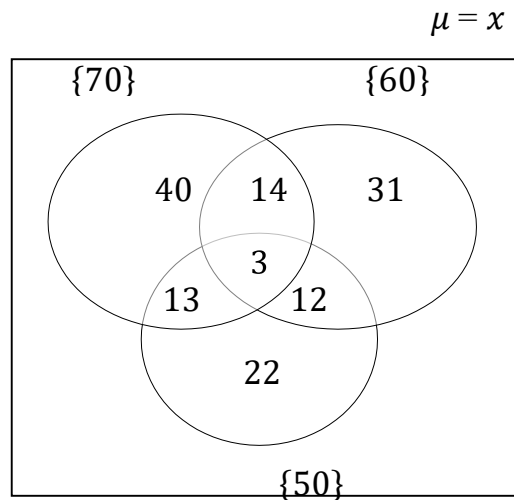
Therefore, those who speak only one language are those who speak French only and those who speak English only.

$$= (30 - 5) + (25 - 5)$$

$$25 + 2 = 4$$

Example 6

A newspaper agent sells three papers: the Times, the Punch, and the Nation. 70 customers buy the Times, 60 the Punch and 50 the Nations. If 17 customers buy both Times and Punch, 15 buy Punch and the Nations and 16 buy the Nation and the Times. How many customers have the agent if 3 customers buy all the papers?

Solution

$n(T)$ = no of customers that buy the Times

$n(P)$ = no of customers that buy the Punch

$n(N)$ = no of customers that buy the Nation.

$\therefore$ Those who buy only times = $70 - [14 + 3 + 13]$

$$= 70 - [30] = 40$$

Those who buy only Punch = $60 - [14 + 3 + 12]$

$$= 60 - [29]$$

$$= 31$$

Those who buy the Nation only = $50 - [13 + 3 + 12]$

$$= 50 - [16 + 12]$$

$$= 50 - 28 = 22$$

$\therefore$ The total number of customers

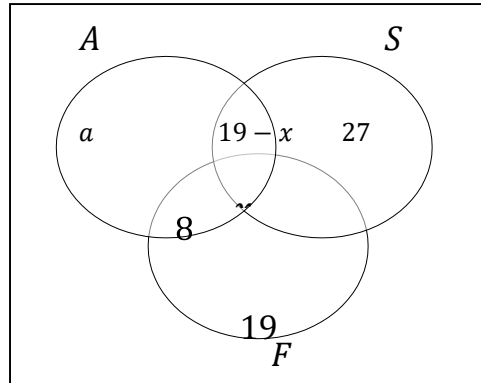
$$= 40 + 14 + 31 + 12 + 22 + 13 + 3 = 135 \text{ Customers.}$$

Example 7

Of the 112 students from the College of Education, 19 read both Accountancy and Sociology, 70 read Accountancy or Sociology but not French, 27 read Sociology, but not Accountancy or French, 53 read Sociology or French, but not Accountancy, 19 read French but not Accountancy or Sociology and 8 read Accountancy and French but not Sociology. Assume that each student reads at least one of these courses.

How many of the students read:

- (i) All three courses?
- (ii) Only one course?
- (iv) Only two courses?



$$\mu = 112$$

$n(A)$ = no of students who read Accountancy

$n(S)$ = no of students who read Sociology

$n(F)$ = no of students who read French

n = no of students who read Accountancy, Sociology and French

i.e. those who read all three courses.

$$n(A \cup S) = 70$$

$$\therefore a + 19 - x + 27 = 70 \quad a - x + 46 = 70$$

$$a = 70 - 46 + x$$

$$a = 24 + x \quad (i)$$

$$n(S \cup F) = 53$$

$$b + 19 + 27 = 53$$

$$b + 46 = 53$$

$$b = 53 - 46 = 7$$

$$\text{Since } n(A \cup S \cup F) = 112$$

$$\therefore a - 19 - x + 8 + x + 19 + b + 27 = 112$$

$$a - 19 - x + 8 + x + 19 + 7 + 27 = 112$$

$$a + 80 = 112 \quad (ii)$$

But from equation (i), $a = 24 + x$

Substitute (i) into (ii) $24 + x + 80 = 112$

$$x + 104 = 112$$

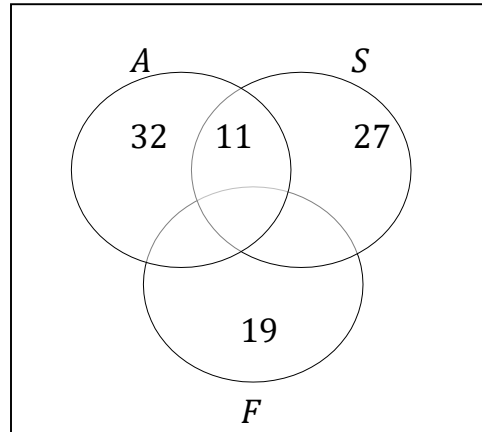
$$x = 112 - 104$$

$$x = 8$$

$$\therefore a = 24 + 8 = 32$$

Then, the Venn diagram is drawn as follows:

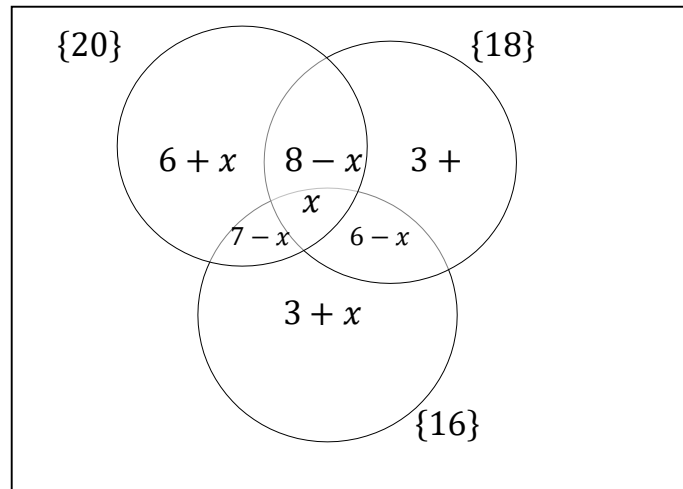
$$\mu = 35$$



- (i) Those who did all three courses = 8
- (ii) Those who did one course = $32 + 27 + 19 = 76$
- (iii) Those who did two courses = $8 + 7 + 11 = 26$

Example 8

In a class of 35 students, 20 speak Hausa, 16 speak Igbo and 18 speak Yoruba. If 8 speak Hausa and Yoruba, 7 speak Hausa and Igbo, and 6 speak Yoruba and Igbo. How many speak all three languages?



$$\text{Those who speak Hausa only} = 20 - [8 - x + x + 6 - x]$$

$$= 20 - [14 - x]$$

$$= 6 + x$$

$$\text{Those who speak Igbo only} = 16 - [6 - x + x + 7 - x]$$

$$= 16 - [13 - x] = 3 + x$$

$$\text{Those who speak Yoruba only} = 18 - [8 - x + x + 7 - x]$$

$$= 18 - [15 - x] = 3 + x$$

$$\therefore H \cup Y \cup I = 50$$

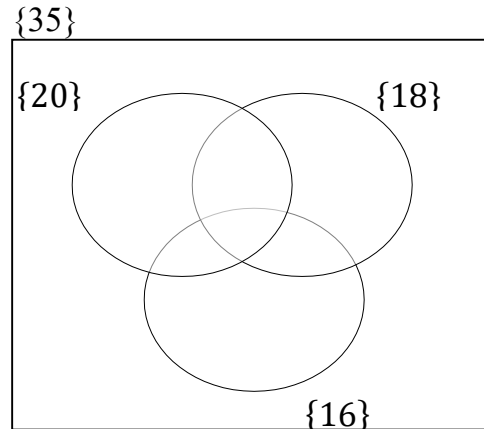
$$6 + x + 8 - x + x + 6 - x + 3 + x + 7 - x + 3 + x = 35$$

$$33 + x = 35$$

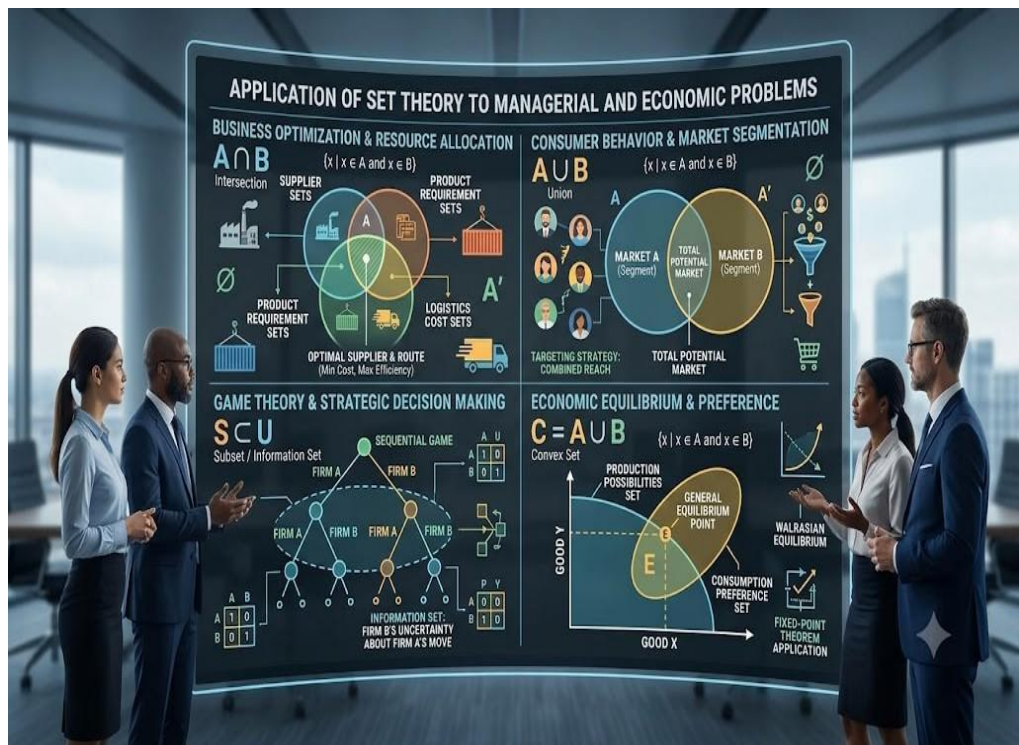
$$x = 35 - 33 = 2$$

$\therefore$ 2 Students speak all three languages.

The Venn diagram can be drawn as thus: -



The roles of Venn diagrams in solving practical word problems involving the interrelationships among elements of a set and sets in a universal set can best be illustrated with Venn diagrams. The use of Venn diagrams is not only a compact way of presenting set information; it is also an alternative approach to solving practical problems involving **sets**.



Source AI generated on 18 July 2026 adopted by Mathias

Scenario 1

At a rapidly growing fintech startup in Lagos, the operations team must navigate the overlap between different customer pain points to optimise their resource allocation. During a high-stress Q3 review, the operations manager groups their struggling active merchants into distinct categories: those experiencing transaction settlement delays, those facing high cross-border processing fees, and those encountering API integration bugs.

By analysing these distinct groups, the management team can pinpoint the critical intersection of all three issues. This reveals a specific segment of high-volume merchants suffering the worst overall experience. Instead of deploying three separate, costly engineering teams to address the problems in isolation, the company uses this overlapping data to make a strategic intervention. They fast-track a single, targeted update to their regional payment gateway, resolving the core issues for this vital user segment first. This targeted approach maximises customer retention whilst minimising unnecessary operational expenditure.

Question

1: Strategic Resource Allocation and Set Intersections

Using the Lagos fintech scenario as a framework, explain how the mathematical concepts of **set intersection** ($\cap$), **union** ($\cup$), and **complements** can be used by management to categorise customer pain points. In your essay, discuss how identifying the intersection of all three merchant issues allows the startup to optimise its resource allocation. Why is addressing this critical intersection more economically efficient than addressing the union of all three problems individually?

2: Economic Decision-Making and Venn Analysis

Imagine you are the operations manager at this Lagos fintech startup. Draft a strategic proposal that uses Venn diagrams and set theory to justify your decision to fast-track a single update to the regional payment gateway. Your essay should address the trade-off between **operational expenditure (OpEx)** and **customer retention**, explaining how partitioning your merchant database into mutually exclusive and overlapping subsets prevents "resource dispersion" and maximises the return on investment (ROI) for engineering interventions.

Self-Assessment Exercise(s) 4

- i. Which of the following statement(s) is/are valid?
 - (b) $A \cup A = A$
 - (c) $A \cap A = A$
 - (d) $A \cup \emptyset = A$

- (e) $A \cup \mu = U$
- (f) $A \cap \emptyset = \emptyset$
- (g) $A \cap \mu = A$
- (h) The complement of A is A^c (i.e. $(A^c)^c = A$)
- ii. What is a Venn diagram? What is its relevance to set theory?
- iii. Draw a Venn diagram illustrating the relationship between sets X and Y if:
 - (a) Some elements are common to X and Y
- iv. If P and Q are sets, using a Venn diagram, shade the following relationship on different diagrams
 - a. $P \cap Q$
 - b. $P \cup Q$



1.7 Summary

The course unit on Set Theory introduced students to the basic concepts and principles for organising and analysing collections of objects, known as sets. The unit began with an introduction to the notion of a set, including its meaning, representation, and types. Students also learned about subsets and the conditions for set equality, which helped them understand relationships among different groups of elements.

The concept of the universal set was studied as the complete set containing all elements under discussion, while different set operations such as union, intersection, complement, and difference were explored through the laws of set operations. Important principles such as the Distributive Law and De Morgan's Law were also examined to simplify and solve set-related problems effectively.

The use of Venn diagrams played a major role in illustrating relationships between sets visually and in solving practical problems involving set operations. In addition, the unit highlighted the application of Set Theory to managerial and economic problems, showing how sets can be used in decision-making, data classification, market analysis, and problem-solving in business and economics.

Overall, the unit provided students with a solid foundation in logical reasoning, analytical thinking, and the practical application of mathematical concepts in real-life situations.



1.8 References/Further Readings/Web Resources

Chiang, A.C. (1984). *Fundamental Methods of Mathematical Economics*. 3rd Edition, McGraw-Hill, Singapore.

Akihito Asano (2012) *An Introduction to Mathematics for Economics*. Cambridge University Press. DOI: <https://doi.org/10.1017/CBO9781139035224>

Karl, J. S. (1990). *Mathematics for Business*. US: Win C. Brown Publishers.

Lucey, T. (1988). *Quantitative Techniques: An Introductory Manual*. (3rd ed.). London: ELBS/DP Publications.



1.9 Possible Answer to Self-Assessment Exercised

Answer to SAEs 1

(!)

Valid Statements: (a), (b), (c), (d), (e), (f), (g), (j)

(!!)

To verify these set theory laws, we will use the following defined sets:

- $A = \{4, 5, 6\}$
- $B = \{3, 4, 6, 7\}$
- $C = \{2, 3, 6\}$

(a) Associative Law

The Associative Law states that the grouping of sets does not affect the result. We can verify this using both **Union** and **Intersection**.

i. Associative Law of Union: $A \cup (B \cup C) = (A \cup B) \cup C$

- **Left-Hand Side (LHS): $A \cup (B \cup C)$**
 - First, find $B \cup C$:
 $B \cup C = \{3, 4, 6, 7\} \cup \{2, 3, 6\} = \{2, 3, 4, 6, 7\}$
 - Now, find $A \cup (B \cup C)$:
 $A \cup (B \cup C) = \{4, 5, 6\} \cup \{2, 3, 4, 6, 7\} = \{2, 3, 4, 5, 6, 7\}$
- **Right-Hand Side (RHS): $(A \cup B) \cup C$**
 - First, find $A \cup B$:

$$A \cup B = \{4, 5, 6\} \cup \{3, 4, 6, 7\} = \{3, 4, 5, 6, 7\}$$

- Now, find $(A \cup B) \cup C$:

$$(A \cup B) \cup C = \{3, 4, 5, 6, 7\} \cup \{2, 3, 6\} = \{2, 3, 4, 5, 6, 7\}$$

$$\text{Since } \text{LHS} = \text{RHS} = \{2, 3, 4, 5, 6, 7\}, \text{ the Associative Law of Union is verified.}$$

ii. Associative Law of Intersection: $A \cap (B \cap C) = (A \cap B) \cap C$

• **Left-Hand Side (LHS): $A \cap (B \cap C)$**

- First, find $B \cap C$:

$$B \cap C = \{3, 4, 6, 7\} \cap \{2, 3, 6\} = \{3, 6\}$$
- Now, find $A \cap (B \cap C)$:

$$A \cap (B \cap C) = \{4, 5, 6\} \cap \{3, 6\} = \{6\}$$

• **Right-Hand Side (RHS): $(A \cap B) \cap C$**

- First, find $A \cap B$:

$$A \cap B = \{4, 5, 6\} \cap \{3, 4, 6, 7\} = \{4, 6\}$$
- Now, find $(A \cap B) \cap C$:

$$(A \cap B) \cap C = \{4, 6\} \cap \{2, 3, 6\} = \{6\}$$

$$\text{Since } \text{LHS} = \text{RHS} = \{6\}, \text{ the Associative Law of Intersection is verified.}$$

(b) Distributive Law

The Distributive Law states that one operation distributes over another. We can verify both versions:

i. Intersection distributes over Union: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

• **Left-Hand Side (LHS): $A \cap (B \cup C)$**

- We already established $B \cup C = \{2, 3, 4, 6, 7\}$.
- Find the intersection with A :

$$A \cap (B \cup C) = \{4, 5, 6\} \cap \{2, 3, 4, 6, 7\} = \{4, 6\}$$

• **Right-Hand Side (RHS): $(A \cap B) \cup (A \cap C)$**

- We already established $A \cap B = \{4, 6\}$.
- Find $A \cap C$:

$$A \cap C = \{4, 5, 6\} \cap \{2, 3, 6\} = \{6\}$$
- Find their union:

$$(A \cap B) \cup (A \cap C) = \{4, 6\} \cup \{6\} = \{4, 6\}$$

$$\text{Since } \text{LHS} = \text{RHS} = \{4, 6\}, \text{ this distributive law is verified.}$$

ii. **Union distributes over Intersection:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- **Left-Hand Side (LHS):** $A \cup (B \cap C)$
 - We already established $B \cap C = \{3, 6\}$.
 - Find the union with A :
 $A \cup (B \cap C) = \{4, 5, 6\} \cup \{3, 6\} = \{3, 4, 5, 6\}$
- **Right-Hand Side (RHS):** $(A \cup B) \cap (A \cup C)$
 - We already established $A \cup B = \{3, 4, 5, 6, 7\}$.
 - Find $A \cup C$:
 $A \cup C = \{4, 5, 6\} \cup \{2, 3, 6\} = \{2, 3, 4, 5, 6\}$
 - Find their intersection:
 $(A \cup B) \cap (A \cup C) = \{3, 4, 5, 6, 7\} \cap \{2, 3, 4, 5, 6\} = \{3, 4, 5, 6\}$
 $\text{Since } \text{LHS} = \text{RHS} = \{3, 4, 5, 6\},$
 $\text{this distributive law is verified.}$

(c) **De Morgan's Law**

De Morgan's Laws require a **Universal Set (\$U\$)** to find complements. Since a universal set is not explicitly specified, the standard practice in this syllabus is to define U as the union of all elements in the given sets:

$$U = A \cup B \cup C = \{2, 3, 4, 5, 6, 7\}$$

We will verify De Morgan's Law for sets A and B :

i. **First Law:** $(A \cup B)' = A' \cap B'$

- **Left-Hand Side (LHS):** $(A \cup B)'$
 - We already established $A \cup B = \{3, 4, 5, 6, 7\}$.
 - Find the complement $(A \cup B)'$ (elements in U but not in $A \cup B$):
 $(A \cup B)' = \{2\}$
- **Right-Hand Side (RHS):** $A' \cap B'$
 - Find A' (elements in U not in A):
 $A' = \{2, 3, 7\}$
 - Find B' (elements in U not in B):
 $B' = \{2, 5\}$
 - Find their intersection:
 $A' \cap B' = \{2, 3, 7\} \cap \{2, 5\} = \{2\}$
 $\text{Since } \text{LHS} = \text{RHS} = \{2\},$ the first De Morgan's Law is verified.

ii. **Second Law:** $(A \cap B)' = A' \cup B'$

- **Left-Hand Side (LHS):** $(A \cap B)'$
 - We already established $A \cap B = \{4, 6\}$.

- Find the complement $(A \cap B)'$ (elements in U but not in $A \cap B$):
 $(A \cap B)' = \{2, 3, 5, 7\}$
- **Right-Hand Side (RHS): $A' \cup B'$**
 - Using $A' = \{2, 3, 7\}$ and $B' = \{2, 5\}$, find their union:
 $A' \cup B' = \{2, 3, 7\} \cup \{2, 5\} = \{2, 3, 5, 7\}$
 $\text{Since } \text{LHS} = \text{RHS} = \{2, 3, 5, 7\}$,
 $\text{the second De Morgan's Law is verified.}$

Answer to SAEs 2

(!)

Venn diagram is a visual and educational tool used to represent the elements of a universal set and show the relationships among objects or finite groups of things. It visually illustrates the similarities and differences between two or more objects, sets, or elements.

Typically, the universal set is represented by a rectangle, and the individual sets within it are represented by circles or oval shapes.

(!!)

Relevance to Set Theory

Venn diagrams play a major and highly relevant role in set theory in several key ways:

- **Visualizing Set Operations:** They provide a clear visual representation of fundamental set operations like the union ($A \cup B$), intersection ($A \cap B$), complements (A' or B'), and differences. For instance, shading different sections of the circles quickly shows which regions satisfy specific algebraic set conditions.
- **Compact Presentation of Information:** Venn diagrams act as a highly compact way of organizing and presenting complex set information and relationships.

Alternative Problem-Solving Approach: Beyond just illustration, Venn diagrams serve as an alternative, structured mathematical approach to solving practical, real-world word problems. They allow you to map out overlapping categories (such as consumers buying different newspapers, or students taking various combinations of courses) and solve for unknown values systematically. Here are the Venn diagrams representing the relationships between sets X and Y under the three specified conditions:

1. (i) Some elements are common to X and Y

This represents an **overlapping** (or intersecting) relationship where the two sets share common elements, meaning $X \cap Y \neq \emptyset$.

- **Explanation:** The circles representing X and Y intersect inside the universal set U . The overlapping region in the middle contains the elements common to both sets ($X \cap Y$).
- (ii) **No element is common to X and Y**
 This represents **disjoint sets** where there is zero overlap between the two groups, meaning $X \cap Y = \emptyset$.
- **Explanation:** Since there are no common elements, the circles representing set X and set Y are drawn completely separate from each other, with no point of contact or intersection.
- (iii) **Y contains every element in X**
 This represents a **subset** relationship where X is a proper subset of Y , written mathematically as $X \subset Y$. Here are the step-by-step solutions to the problems based on the rules of set theory:
- (a) **Element Listing and General Venn Diagram**
 First, let's identify the elements of each set based on the descriptions:
- **Set A (Prime numbers between 10 and 30):**
 $A = \{11, 13, 17, 19, 23, 29\}$
 - **Set B (Even numbers between 9 and 29):**
 $B = \{10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$
 - **Set C (Multiples of 3 between 10 and 29):**
 $C = \{12, 15, 18, 21, 24, 27\}$
- To draw the Venn diagram, let's look at their intersections (shared elements):
- **Intersection of A and B ($A \cap B$):** There are no even prime numbers between 10 and 30. Therefore, $A \cap B = \emptyset$ (Disjoint sets).
 - **Intersection of A and C ($A \cap C$):** There are no prime numbers between 10 and 30 that are multiples of 3. Therefore, $A \cap C = \emptyset$ (Disjoint sets).
 - **Intersection of B and C ($B \cap C$):** These are numbers that are both even and multiples of 3 (i.e., multiples of 6) within the given range:
 $B \cap C = \{12, 18, 24\}$
 - **Diagram Layout:** Set A sits completely isolated as its own circle since it shares no elements with B or C . Circles B and C overlap, with $\{12, 18, 24\}$ in the shared middle section.
- (b) **Regions Shaded in Separate Venn Diagrams**
- (i) **Region for $A \cup (B \cap C)$**
 This region includes all of set A plus the overlapping section where B and C intersect.

(ii) Region for $(A \cap B) \cup C$

Since $A \cap B = \emptyset$ (nothing is shared between A and B), the intersection region contains no elements. Therefore, this expression simplifies to just the entire region of set C .

(c) Listing the Elements of b(i) and b(ii)**Elements of b(i): $A \cup (B \cap C)$**

- We know $A = \{11, 13, 17, 19, 23, 29\}$ and $B \cap C = \{12, 18, 24\}$.
- Uniting them together yields:
 $A \cup (B \cap C) = \{11, 12, 13, 17, 18, 19, 23, 24, 29\}$

Elements of b(ii): $(A \cap B) \cup C$

- We know $A \cap B = \emptyset$ and $C = \{12, 15, 18, 21, 24, 27\}$.
- Uniting the empty set with C yields:
 $(A \cap B) \cup C = \{12, 15, 18, 21, 24, 27\}$

(d) Evaluation and Responsibility

- **Question:** Is $A \cup (B \cap C) = (A \cap B) \cup C$?
- **Answer: No**, they are not equal. As shown in part (c), the elements of the two sets are completely different.

What is responsible for this?

This inequality is caused by the **absence of the Associative Law across mixed operations**.

1. **Operation Mismatch:** The associative property (moving brackets around) only holds true when the operators are identical throughout the entire expression (e.g., $A \cup (B \cup C) = (A \cup B) \cup C$). When operations are mixed (combining both a Union $\cup$ and an Intersection $\cap$), you cannot simply shift parentheses.
2. **Distributive Law Requirement:** Changing the position of the brackets fundamentally transforms the algebraic meaning of the expression. To accurately expand a mixed expression like $A \cup (B \cap C)$, you must apply the **Distributive Law** instead:
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 Because $(A \cap B) \cup C$ follows an entirely different logical distribution, the final elements and regions are fundamentally distinct.

Answer to SAEs 3

Here are the step-by-step solutions to the problems based on the rules of set theory:

(a) Element Listing and General Venn Diagram

First, let's identify the elements of each set based on the descriptions:

- **Set A (Prime numbers between 10 and 30):**
 $A = \{11, 13, 17, 19, 23, 29\}$
- **Set B (Even numbers between 9 and 29):**
 $B = \{10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$
- **Set C (Multiples of 3 between 10 and 29):**
 $C = \{12, 15, 18, 21, 24, 27\}$

To draw the Venn diagram, let's look at their intersections (shared elements):

- **Intersection of A and B ($A \cap B$):** There are no even prime numbers between 10 and 30. Therefore, $A \cap B = \emptyset$ (Disjoint sets).
- **Intersection of A and C ($A \cap C$):** There are no prime numbers between 10 and 30 that are multiples of 3. Therefore, $A \cap C = \emptyset$ (Disjoint sets).
- **Intersection of B and C ($B \cap C$):** These are numbers that are both even and multiples of 3 (i.e., multiples of 6) within the given range:
 $B \cap C = \{12, 18, 24\}$
- **Diagram Layout:** Set A sits completely isolated as its own circle since it shares no elements with B or C. Circles B and C overlap, with $\{12, 18, 24\}$ in the shared middle section.

(b) Regions Shaded in Separate Venn Diagrams

(i) Region for $A \cup (B \cap C)$

This region includes all of set A plus the overlapping section where B and C intersect.

(ii) Region for $(A \cap B) \cup C$

Since $A \cap B = \emptyset$ (nothing is shared between A and B), the intersection region contains no elements. Therefore, this expression simplifies to just the entire region of set C.

(c) Listing the Elements of b(i) and b(ii)

Elements of b(i): $A \cup (B \cap C)$

- We know $A = \{11, 13, 17, 19, 23, 29\}$ and $B \cap C = \{12, 18, 24\}$.
- Uniting them together yields:
 $A \cup (B \cap C) = \{11, 12, 13, 17, 18, 19, 23, 24, 29\}$

Elements of b(ii): $(A \cap B) \cup C$

- We know $A \cap B = \emptyset$ and $C = \{12, 15, 18, 21, 24, 27\}$.
- Uniting the empty set with C yields:
 $(A \cap B) \cup C = \{12, 15, 18, 21, 24, 27\}$

(d) Evaluation and Responsibility

- **Question:** Is $A \cup (B \cap C) = (A \cap B) \cup C$?

Answer:

- **No**, they are not equal. As shown in part (c), the elements of the two sets are completely different.

What is responsible for this?

This inequality is caused by the **absence of the Associative Law across mixed operations**.

1. **Operation Mismatch:** The associative property (moving brackets around) only holds true when the operators are identical throughout the entire expression (e.g., $A \cup (B \cup C) = (A \cup B) \cup C$). When operations are mixed (combining both a Union $\cup$ and an Intersection $\cap$), you cannot simply shift parentheses.
2. **Distributive Law Requirement:** Changing the position of the brackets fundamentally transforms the algebraic meaning of the expression. To accurately expand a mixed expression like $A \cup (B \cap C)$, you must apply the **Distributive Law** instead:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Because $(A \cap B) \cup C$ follows an entirely different logical distribution, the final elements and regions are fundamentally distinct.

Here are the descriptions and the corresponding Venn diagrams representing each relationship between sets P and Q inside a universal set U .

(a) $P \cap Q$ (The intersection of P and Q)

This represents the set containing elements that belong to **both** P and Q simultaneously.

- **Shading:** Only the overlapping (middle) region where circles P and Q intersect is shaded.

(b) $P \cup Q$ (The union of P and Q)

This represents the set containing elements that belong to P , or Q , or to both.

- **Shading:** The entire area of circle P , circle Q , and their intersection is completely shaded.

(c) P' (The complement of P)

Also written as P^c or P^1 , this represents the set of all elements in the universal set U that are **not** in set P .

- **Shading:** Everything outside circle P is shaded. This includes the region of the universal set U as well as the part of circle Q

that does not overlap with P . Circle P is left completely unshaded.

(d) Q' (The complement of Q)

Also written as Q^c or Q^1 , this represents the set of all elements in the universal set U that are **not** in set Q .

- **Shading:** Everything outside circle Q is shaded. This includes the region of the universal set U as well as the part of circle P that does not overlap with Q . Circle Q is left completely unshaded.